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The Existence of Three Positive Solutions for p-Laplacian Difference Equation with Delay*

Wang Lin-jun[1,2] and Gong Cheng-chun[2]
(*1. Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013*)
(*2. School of Mathematics, Jilin University, Changchun, 130012*)

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Abstract: In this paper, we study the multiplicity of positive solutions for a class of p-Laplacian difference equations with delay. We propose sufficient conditions for the existence of at least three positive solutions and we also provide two numerical examples to illustrate the theoretical results.

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1 Introduction

The p-Laplacian differential equations have been vastly applied in many fields such as non-Newtonian mechanics, economics, neural networks, ecology, nonlinear flow laws, etc. (see [1–5]).

One of the important examples was described in [6]. Let $x = (x_1, x_2)$ be the two Cartesian coordinates in the plane of the glacier occupying a Lipschitzian domain Ω. We denote by $u(x)$ the horizontal velocity component of the ice at the point x. After a rescaling of the physical velocity of the ice, u satisfies the equation

$$-\mathrm{div}(\psi(|\nabla u|)\nabla u) = e \qquad \text{in } \Omega, \tag{1.1}$$

where e is a hydrostatic pressure force acting on the glacier and ψ is a function resulting from a constitutive law for the ice. The typical case of (1.1) is the following equation:

$$\mathrm{div}(\phi_p(\nabla u)) + b(|x|)g(u) = 0, \qquad x \in B(0, R_2)/B(0, R_1), \tag{1.2}$$

where $\phi_p(s) = |s|^{p-2}s$ with $p > 1$ and $B(0, R_i) \subset \mathbf{R}^d$ are the open balls centred about the origin with radius R_i, respectively. People are interested in considering positive radial solutions of the equation (1.2). Then (1.2) can be reduced to the following form (see [7]):

$$r^{1-d}(r^{d-1}\phi_p(u'))' + b(r)g(u(r)) = 0, \qquad R_1 < r < R_2. \tag{1.3}$$

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Let

$$s = -\int_r^{R_2} t^{-\frac{d-1}{p-1}} \mathrm{d}t, \qquad v(s) = u(r(s)), \qquad \rho = -\int_{R_1}^{R_2} t^{-\frac{d-1}{p-1}} \mathrm{d}t.$$

The equation (1.3) is transformed into

$$(\phi_p(v'(s)))' + r^{\frac{p(d-1)}{p-1}}(s) b(r(s)) g(v(s)) = 0, \qquad \rho < s < 0. \tag{1.4}$$

With variable change $t = 1 - \dfrac{s}{\rho}$ and $y(t) = v(s)$, the equation (1.4) reads as

$$(\phi_p(y'(t)))' + c(t) g(y(t)) = 0, \qquad 0 < t < 1, \tag{1.5}$$

where

$$c(t) = (-\rho)^p r^{\frac{p(d-1)}{p-1}}[\rho(1-t)] b[r(\rho(1-t))].$$

The equation (1.5) is a typical type of one-dimensional p-Laplacian equation.

In the real world, some processes are more reasonably described as p-Laplacian differential equations with delay (see [2, 8–9]). The reason is that the differential of the unknown solutions depends not only on the values of the unknown solutions at the current time, but also on the values prior to that. Such equations, to a certain extent, reflect much more exactly the physical reality than the equations without delay.

In recent years, p-Laplacian differential equations with delay have received a lot of attention (see [8–9]). There exists a large number of papers devoted to study the existence of positive solutions for such problems (see [1–2], [8–9]).

In reality, the equation (1.5) is applied together with some boundary value conditions. We can apply the standard Euler method to discretize the equation (1.5) and approximate its solutions numerically. An immediate and natural question is if the corresponding difference equation together with boundary conditions has positive solutions.

In this paper, we are concerned with the following p-Laplacian difference equation with delay. We prove the multiplicity of positive solutions for the following system of equations:

$$\Delta(\phi_p(\Delta u(n-1))) + a(n) f(n, u(n-t_0)) = 0, \qquad n \in \{1, 2, \cdots, N\}, \tag{1.6}$$

$$\Delta u(N) = 0, \qquad u(n) = 0, \quad n \in \{-t_0, -t_0+1, \cdots, 0\}, \tag{1.7}$$

where

$$\Delta u(n) = u(n+1) - u(n)$$

and t_0 is a positive integer.

A sequence $\{u(n)\}_{n=-t_0}^{N+1}$ is said to be a positive solution of the problem (1.6)-(1.7), if it satisfies (1.6)-(1.7) with $u(n) > 0$, $n \in \{1, 2, \cdots, N+1\}$.

Recently, the existence of positive solutions for p-Laplacian difference equations with different types of boundary value conditions is investigated in [3, 10–15] and the references therein.

By using Guo-Krasnoselskii fixed point theorem and a fixed point index theorem, He[16] proved the existence of one or two positive solutions for the following system of p-Laplacian difference equations:

$$\Delta(\phi_p(\Delta u(n-1))) + a(n) f(u(n)) = 0, \qquad n \in \{1, 2, \cdots, N\}, \tag{1.8}$$

$$\Delta u(0) = u(N+1) = 0. \tag{1.9}$$