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A Smoothing SAA Method for a Stochastic Linear Complementarity Problem

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Abstract: Utilizing the well-known aggregation technique, we propose a smoothing sample average approximation (SAA) method for a stochastic linear complementarity problem, where the underlying functions are represented by expectations of stochastic functions. The method is proved to be convergent and the preliminary numerical results are reported.

Key words: aggregation technique, smoothing SAA method, stochastic linear complementarity problem

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1 Introduction

In this paper, we consider the following stochastic linear complementarity problem (SLCP): find $\boldsymbol{x} \in \mathbf{R}^n$ such that

$$\boldsymbol{x} \geq 0, \quad \Psi(\boldsymbol{x}) \geq 0, \quad \Psi(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{x} = 0, \quad \Psi(\boldsymbol{x}) = \mathbb{E}[M(\xi(\omega))\boldsymbol{x} + q(\xi(\omega))], \tag{1.1}$$

where $M(\cdot) : \mathbf{R}^k \to \mathbf{R}^{n\times n}$ and $q(\cdot) : \mathbf{R}^k \to \mathbf{R}^n$ are random mappings, $\boldsymbol{\xi} : \Omega \to \Xi \subset \mathbf{R}^k$ is a random vector defined on probability space $(\Omega, \mathcal{F}, \mathrm{P})$, and $\mathbb{E}$ denotes the mathematical expectation. Throughout the paper, we assume that $M(\xi(\omega))$ and $q(\xi(\omega))$ are measurable functions of ω satisfying

$$\mathbb{E}\left[\|M(\xi(\omega))\|^2 + \|q(\xi(\omega))\|^2\right] < \infty.$$

To ease the notation, we write $\xi(\omega)$ as $\boldsymbol{\xi}$ and this should be distinguished from $\boldsymbol{\xi}$ being a deterministic vector of Ξ in a context.

SLCP (1.1) is a natural extension of the deterministic complementarity problem and can be seen as a special case of the stochastic variational inequality problem which was first

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proposed by Gürkan *et al.*[1] Over the past several decades, the complementarity problem has been intensively studied for its extensively application in engineering, economics, game theory and networks (see [2]). While in practical, there are some important instances that the problem data contains some uncertain factors, and consequently, the stochastic complementarity models are proposed to reflect the uncertainties. Some examples of the stochastic complementarity problem, arising from the areas of economics, engineering and operations management can be found in [3].

In this paper, we focus on numerical methods for solving (1.1). Evidently, if the integral involved in the mathematical expectation problems exists or is computable, then the problem (1.1) is reduced to the usual LCP problem and the existing methods in [2] can be applied directly to it. However, in many cases, an exact evaluation of the expected value in (1.1) for $\boldsymbol{x}$ is either impossible or prohibitively expensive. The sample average approximation (SAA) method is suggested to handle this difficulty (see [4–6]). The basic idea of SAA is to generate an independent identically distributed (iid) sample $\boldsymbol{\xi}^1, \cdots, \boldsymbol{\xi}^N$ of $\boldsymbol{\xi}$, and then approximate the expected value with a sample average. In this context, SLCP (1.1) is approximated by

$$\hat{\Psi}^N(\boldsymbol{x}) \geq 0, \quad \boldsymbol{x} \geq 0, \qquad \hat{\Psi}^N(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{x} = 0, \quad \text{w.p.1}, \tag{1.2}$$

where

$$\hat{\Psi}^N(\boldsymbol{x}) = \frac{1}{N}\sum_{i=1}^{N}\left[M(\boldsymbol{\xi}^i)\boldsymbol{x} + q(\boldsymbol{\xi}^i)\right] \tag{1.3}$$

is a sample-average mapping of $\Psi(\boldsymbol{x})$. We refer to (1.1) as a true problem and (1.2) as an SAA problem to (1.1).

Recently, Chen and Fukushima[7] consider another type of stochastic linear complementarity problem:

$$\boldsymbol{x} \geq 0, \quad M(\xi(\omega))\boldsymbol{x} + q(\xi(\omega)) \geq 0, \quad [M(\xi(\omega))\boldsymbol{x} + q(\xi(\omega))]^{\mathrm{T}}\boldsymbol{x} = 0, \qquad \text{a.e. } \omega \in \Omega. \tag{1.4}$$

They formulate (1.4) as a problem of minimizing an expected residual defined by an NCP function, which is referred to as the ERM method. Then, they employ a qusi-Monte Carlo method and give some convergence results under suitable assumptions on the associated matrices.

In this paper, inspired by ERM method, incorporating SAA method with the well known aggregation function, we propose a smoothing SAA method for solving (1.1). We study the almost sure existence of solutions of SAA problem when the sample size is sufficiently large and show that under moderate conditions, a sequence of SAA solutions converges to the solution of counterpart true problem with probability one at exponential rate as the sample size tends to infinity. Finally, some numerical results are also reported.

Throughout this paper we use the following notations. Let $\|\cdot\|$ denote the Euclidean norm of a vector or the Frobenius norm of a matrix and

$$\mathrm{dist}(\boldsymbol{x}, D) := \inf_{\boldsymbol{x}' \in D}\|\boldsymbol{x} - \boldsymbol{x}'\|$$

denote the distance from a point $\boldsymbol{x}$ to a set D. Let $\mathbb{B}$ be the closed unite ball and $\mathbb{B}(\boldsymbol{x}, \delta)$ be the closed ball around $\boldsymbol{x}$ of radius $\delta > 0$. For two sets $A, C \subset \mathbf{R}^n$, we denote by

$$\mathbb{D}(A, C) := \inf\{t > 0 : A \subset C + t\,\mathbb{B}\}$$