Communications
in
Mathematical
Research
29(4)(2013), 297–304

A New Kind of Iteration Method for Finding Approximate Periodic Solutions to Ordinary Differential Equations

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Communicated by Li Yong

Abstract: In this paper, a new kind of iteration technique for solving nonlinear ordinary differential equations is described and used to give approximate periodic solutions for some well-known nonlinear problems. The most interesting features of the proposed methods are its extreme simplicity and concise forms of iteration formula for a wide range of nonlinear problems.

Key words: iteration method, approximate periodic solution, ordinary differential equation

2000 MR subject classification: 35A15, 34A34, 34B15, 34C15

Document code: A

Article ID: 1674-5647(2013)04-0297-08

1 Introduction

In this paper we deals with the approximate solutions to the equations of the following form

$$u'' + a(t)u' + b(t)u = f(t,\ u,\ u'), \tag{1.1}$$

where $a(\,\cdot\,)$, $b(\,\cdot\,)$ and $f(\,\cdot\,,\ u,\ u')$ are T-periodic in t.

From the point of view of physical applications, as well as from the point of view of theory, it is very important to know how to solve the equation (1.1).

Inspired by the constant variation formula, we present a new kind of iteration method for finding approximate solutions of the equation above. The method will be shown to solve a large class of nonlinear problems effectively, easily and accurately.

Before providing our iteration procedure, two other nonlinear analytical methods: "variational iteration method (VIM)" developed by He in [1–8] and "Adomian decomposition

Received date: Dec. 10, 2009.
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method (ADM)" developed by Adomian in [9–10] are mentioned. The two methods, which accurately compute the solutions in a series form or in an extreme form, are of great interesting to applied sciences. Wazwaz[11] raised the method FOR to detail the VIM and ADM, and a comparison between them. In particular, the issue 207 of Journal of Computational and Applied Mathematics consists of a collection of recently obtained results and various new interpretation of earlier conclusions pertinent to the application of the VIM for real-life nonlinear problems.

Our method seems more efficient than VIM and ADM. Furthermore, there is no any secular term in the computation.

The layout of the paper is as follows: In Section 2, we give the main idea of the method and the iteration procedure. In Section 3, several examples are given to illustrate the effectiveness of our method.

2 Iteration Procedure

Consider the equation (1.1). Let $\varphi_1(t)$, $\varphi_2(t)$ be two fundamental solutions with $\varphi_1(0) = 1$, $\varphi_1'(0) = 0$, $\varphi_2(0) = 0$, $\varphi_2'(0) = 1$. Set

$$U(t) = \begin{pmatrix} \varphi_1(t) & \varphi_2(t) \\ \varphi_1'(t) & \varphi_2'(t) \end{pmatrix}, \qquad U(0) = I.$$

Assume that $x(t)$ is a T-periodic solution of the equation (1.1). Then

$$\begin{pmatrix} x(t) \\ x'(t) \end{pmatrix} = U(t) \left(\begin{pmatrix} x(0) \\ x'(0) \end{pmatrix} + \int_0^t U^{-1}(s) \begin{pmatrix} 0 \\ f(s,\ x(s),\ x'(s)) \end{pmatrix} \mathrm{d}s \right). \tag{2.1}$$

Since

$$\begin{pmatrix} x(0) \\ x'(0) \end{pmatrix} = \begin{pmatrix} x(T) \\ x'(T) \end{pmatrix},$$

one has

$$\begin{pmatrix} x(0) \\ x'(0) \end{pmatrix} = -\left(U(T) - I\right)^{-1} U(T) \int_0^T U^{-1}(s) \begin{pmatrix} 0 \\ f(s,\ x(s),\ x'(s)) \end{pmatrix} \mathrm{d}s, \tag{2.2}$$

provided $U(T) - I$ is non-singular.

Now substituting (2.2) into (2.1), we have

$$\begin{pmatrix} x(t) \\ x'(t) \end{pmatrix} = U(t) \left(-\left(U(T) - I\right)^{-1} U(T) \int_0^T U^{-1}(s) \begin{pmatrix} 0 \\ f(s,\ x(s),\ x'(s)) \end{pmatrix} \mathrm{d}s \right.$$
$$\left. + \int_0^t U^{-1}(s) \begin{pmatrix} 0 \\ f(s,\ x(s),\ x'(s)) \end{pmatrix} \mathrm{d}s \right).$$

Let

$$\varPhi(t) = U(t) \left(-\left(U(T) - I\right)^{-1} U(T) \right), \qquad \varPhi_1(t) = U(t).$$