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Some Notes on Normality Criteria of Meromorphic Functions

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Abstract: In this paper, we study the normality of families of meromorohic functions related to a Hayman conjecture. We prove that the conditions in Hayman conjecture and in other criterions can be relaxed. The results in this paper improve some previous results.

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1 Introduction and Main Results

We use $\mathbb{C}$ to denote the open complex plane, $\hat{\mathbb{C}}(= \mathbb{C} \cup \{\infty\})$ to denote the extended complex plane and D to denote a domain in $\mathbb{C}$. A family $\mathcal{F}$ of meromorphic functions defined in $D \subset \mathbb{C}$ is said to be normal, if any sequence $\{f_n\} \subset \mathcal{F}$ contains a subsequence which converges spherically, and locally, uniformly in D to a meromorphic function or ∞. Clearly, $\mathcal{F}$ is said to be normal in D if and only if it is normal at every point in D (see [1]).

Let D be a domain in $\mathbb{C}$, f and g be two meromorphic functions, a and b be complex numbers. If $g(z) = b$ whenever $f(z) = a$, we write

$$f(z) = a \Rightarrow g(z) = b.$$

If $f(z) = a \Rightarrow g(z) = b$ and $g(z) = b \Rightarrow f(z) = a$, we write

$$f(z) = a \Leftrightarrow g(z) = b.$$

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According to Bloch's principle (see [2]), every condition which reduces a meromorphic function in the plane $\mathbb{C}$ to a constant forces a family of meromorphic functions in a domain D normal. Although the principle is false in general (see [3]), many authors proved normality criterion for families of meromorphic functions by starting from Liouville-Picard type theorem (see [4]). Moreover, it is interesting to find normality criteria from the point of view of shared values. Schwick[5] first proved an interesting result that a family of meromorphic functions in a domain is normal if every function in that family shares three distinct finite complex numbers with its first derivative. And later, more results about normality criteria concerning shared values have emerged. In recent years, this subject has attracted the attention of many researchers worldwide.

In this paper, we use $\sigma(x, y)$ to denote the spherical distance between x and y and the definition of the spherical distance can be found in [6].

Theorem 1.1[7] *Let $\mathcal{F}$ be a family of meromorphic functions in the unit disc Δ, a and b be distinct complex numbers, and c be a nonzero complex number. If for every $f \in \mathcal{F}$, $f(z) = 0 \Leftrightarrow f'(z) = a$ and $f(z) = c \Leftrightarrow f'(z) = b$, then $\mathcal{F}$ is normal in Δ.*

In 2004, Singh A P and Singh A[8] proved that the condition for the constants in Theorem 1.1 to be the same for all $f \in \mathcal{F}$ can be relaxed to some extent, and they proved the following theorem.

Theorem 1.2[8] *Let $\mathcal{F}$ be a family of meromorphic functions in the unit disc Δ. For each $f \in \mathcal{F}$, suppose that there exist nonzero complex numbers b_f, c_f satisfying*:

(i) $\dfrac{b_f}{c_f}$ *is a constant*;

(ii) $\min\{\sigma(0, b_f), \sigma(0, c_f), \sigma(b_f, c_f)\} \geq m$ *for some* $m > 0$;

(iii) $f(z) = 0 \Leftrightarrow f'(z) = 0$ *and* $f(z) = c_f \Leftrightarrow f'(z) = b_f$.

Then $\mathcal{F}$ is normal in Δ.

Theorem 1.3[9] *Let $\mathcal{F}$ be a family of holomorphic* (*meromorphic*) *functions in a domain D, $n \in N$, $a \neq 0$, and $b \in \mathbb{C}$. If $f'(z) - af^n(z) \neq b$ for each function $f \in \mathcal{F}$ and $n \geq 2$ $(n \geq 3)$, then $\mathcal{F}$ is normal in D.*

From the idea of Theorem 1.2, we generalize Theorem 1.3 as the following theorem.

Theorem 1.4 (Main Theorem I) *Let $\mathcal{F}$ be a family of meromorphic functions in the unit disc Δ, and $n(\geq 3)$ be a positive integer. For every $f \in \mathcal{F}$, there exist finite nonzero complex numbers b_f, c_f depending on f satisfying*:

(i) $\dfrac{b_f}{c_f}$ *is a constant*;

(ii) $\min\{\sigma(0, b_f), \sigma(0, c_f), \sigma(b_f, c_f)\} \geq m$ *for some* $m > 0$;

(iii) $f'(z) - \dfrac{1}{b_f^{n-1}} f^n(z) \neq c_f$.

Then $\mathcal{F}$ is normal in Δ.