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Complete Convergence of Weighted Sums for Arrays of Rowwise m-negatively Associated Random Variables

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Abstract: In this paper, we discuss the complete convergence of weighted sums for arrays of rowwise m-negatively associated random variables. By applying moment inequality and truncation methods, the sufficient conditions of complete convergence of weighted sums for arrays of rowwise m-negatively associated random variables are established. These results generalize and complement some known conclusions.

Key words: complete convergence, negatively associated, m-negatively associated, weighted sum

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1 Introduction

Let $\{X_n,\ n \geq 1\}$ be a sequence of random variables. Hsu and Robbins[1] introduced the concept of complete convergence of $\{X_n\}$. A sequence $\{X_n,\ n = 1, 2, \cdots\}$ of random variables is said to converge completely to a constant C if

$$\sum_{n=1}^{\infty} P(|X_n - C| > \epsilon) < \infty, \qquad \epsilon > 0.$$

In view of the Borel-Cantelli lemma, this implies that $X_n \to C$ almost surely. The converse is true if $\{X_n,\ n \geq 1\}$ is a sequence of independent random variables.

Definition 1.1 *A finite family of random variables $\{X_i,\ 1 \leq i \leq n\}$ is said to be negatively associated (NA, for short) if for every pair of disjoint subsets A and B of $\{1, 2, \cdots, n\}$*

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and any real nondecreasing coordinate-wise functions f_1 *on* $\mathbf{R}^A$ *and* f_2 *on* $\mathbf{R}^B$

$$\mathrm{cov}(f_1(X_i,\ i \in A),\ f_2(X_i,\ i \in B)) \le 0$$

whenever f_1 *and* f_2 *are such that covariance exists.*

An infinite family of random variables $\{X_i, -\infty < i < \infty\}$ *is NA if every finite subfamily is NA.*

The definition of NA was introduced by Alam and Saxena[2] and was studied by Joag-Dev *et al.* (see [3–4]). As pointed out and proved by Joag-Dev and Proschan[3], a number of well-known multivariate distributions possess the NA property. Negative association has found important and wide applications in multivariate statistical analysis and reliability. Many investigators have discussed applications of negative association to probability, stochastic processes and statistics.

Definition 1.2 *Let* $m \ge 1$ *be a fixed integer. A sequence of random variables* $\{X_i,\ i \ge 1\}$ *is said to be m-negatively associated (m-NA, for short) if for any* $n \ge 2$ *and* $i_1, i_2, \cdots, i_n$ *such that* $|i_k - i_j| \ge m$ *for all* $1 \le k \ne j \le n$, $\{X_{i_1},\ X_{i_2},\ \cdots,\ X_{i_n}\}$ *is NA.*

The m-NA random variables is a natural extension from NA random variables. Actually, the NA sequence is just the 1-NA sequence. Moreover, Hu *et al.*[5] showed that there exists a sequence which is not NA but 2-NA.

Hu *et al.*[6] proved a very general result for complete convergence of rowwise independent arrays of random variables which is stated in Theorem 1.1.

Theorem 1.1[6] *Let* $\{X_{ni},\ 1 \le i \le k_n,\ n \ge 1\}$ *be an array of rowwise independent arrays of random variables. Suppose that for every* $\epsilon > 0$ *and some* $\delta > 0$,

(i) $\sum\limits_{n=1}^{\infty} c_n \sum\limits_{i=1}^{k_n} P\{|X_{ni}| > \epsilon\} < \infty$;

(ii) *there exists a* $j \ge 2$ *such that* $\sum\limits_{n=1}^{\infty} c_n \Big(\sum\limits_{i=1}^{k_n} E|X_{ni}|^2 I(|X_{ni}| \le \delta)\Big)^{j/2} < \infty$;

(iii) $\sum\limits_{i=1}^{k_n} EX_{ni} I(|X_{ni}| \le \delta) \to 0$ *as* $n \to \infty$.

Then

$$\sum_{n=1}^{\infty} c_n P\bigg\{\bigg|\sum_{i=1}^{k_n} X_{ni}\bigg| > \epsilon\bigg\} < \infty, \qquad \epsilon > 0.$$

Hu *et al.*[7] obtained the complete convergence of maximum partial sums for arrays of rowwise NA random variables by using an exponential inequality obtained by Shao[8] and their result is given in Theorem 1.2.

Theorem 1.2[7] *Let* $\{X_{ni},\ 1 \le i \le k_n,\ n \ge 1\}$ *be an array of rowwise NA random variables such that the conditions* (i) *and* (ii) *in Theorem* 1.1 *are satisfied. Then*

$$\sum_{n=1}^{\infty} c_n P\bigg\{\max_{1 \le k \le k_n} \bigg|\sum_{i=1}^{k} (X_{ni} - EX_{ni} I(|X_{ni}| \le \delta))\bigg| > \epsilon\bigg\} < \infty, \qquad \epsilon > 0.$$