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Multilinear Commutators of Sublinear Operators on Triebel-Lizorkin Spaces

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Abstract: In this paper, the boundedness of multilinear commutators related to sublinear operators with Lipschitz function on Triebel-Lizorkin spaces is given. As an application, we prove that the multilinear commutators of Littlewood-Paley operator and Bochner-Riesz operator are bounded on Triebel-Lizorkin spaces.

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1 Introduction

In 2002, Pérez and Trujillo-González[1] introduced a kind of multilinear commutators of singular integral operators with $Osc_{\exp L^r}$ $(r \geq 1)$ function and obtained sharp weighted estimates for this kind of multilinear commutators. Since then, the properties of multilinear commutators have been widely studied in harmonic analysis (see [1–10]). Hu *et al.*[2], Meng and Yang[3] proved the boundedness of multilinear commutators with non-doubling measures. Chen and Ma[4] established that multilinear commutators related to Calderón-Zygmund operator and fractional integral operator with Lipschitz function are bounded in Triebel-Lizorkin spaces. Meanwhile, weighted weak-type estimates for multilinear commutators of fractional integrals on homogeneous type spaces were discussed by Gorosito *et al.*[5]. Later, Mo and Lu[6] studied the boundedness for multilinear commutators of Marcinkiewicz

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integral operators on Triebel-Lizorkin spaces and Hardy spaces. Recently, the weighted estimates for multilinear commutators of Littlewood-Paley operators and Marcinkiewicz integrals were established by Xue and Ding[7] and Zhang[8], respectively. The bounded properties for the multilinear commutators of θ-type Calderón-Zygmund operators were considered by authors in [9]. In 2011, Xie *et al.*[10] established the endpoint estimate for multilinear commutators of Bochner-Riesz operators. In this paper, the boundedness of multilinear commutators related to sublinear operators on Triebel-Lizorkin spaces is considered. And as an application, we obtain that the multilinear commutators of Littlewood-Paley operator and Bochner-Riesz operator are bounded on Triebel-Lizorkin spaces.

Throughout this paper, C always means a constant independent of the main parameters involved, but it may be different from line to line. Q denote a cube of $\mathbf{R}^n$ with side parallel to the axes, and for a cube Q and a locally integrable function f, let $f_Q = |Q|^{-1}\int_Q f(x)\mathrm{d}x$. For $\beta > 0$ and $p > 1$, let $\dot{F}_p^{\beta,\infty}$ be the homogeneous Triebel-Lizorkin space, and the Lipschitz space $\dot{\Lambda}_\beta$ is the space of functions f such that

$$\|f\|_{\dot{\Lambda}_\beta} = \sup_{x,h\in\mathbf{R}^n;h\neq 0} \frac{|\varDelta_h^{[\beta]+1} f(x)|}{|h|^\beta} < \infty,$$

where $\varDelta_h^k$ denotes the k-th difference operator (see [11]).

Given any positive integer m, for $1 \le i \le m$, we denote by C_i^m the family of all finite subsets $\sigma = \{\sigma(1), \sigma(2), \cdots, \sigma(i)\}$ of $\{1, 2, \cdots, m\}$ with i different elements. For any $\sigma \in C_i^m$, the complementary sequences σ' is given by $\sigma' = \{1, 2, \cdots, m\}\backslash\sigma$. Let $\boldsymbol{b} = (b_1, b_2, \cdots, b_m)$ be a finite family of locally integrable functions. For all $1 \le i \le m$ and $\sigma = \{\sigma(1), \cdots, \sigma(i)\} \in C_i^m$, we denote $\boldsymbol{b}_\sigma = (b_{\sigma(1)}, \cdots, b_{\sigma(i)})$ and the product $b_\sigma = b_{\sigma(1)}\cdots b_{\sigma(i)}$. If $\beta_{\sigma(1)}+\cdots+\beta_{\sigma(i)} = \beta_\sigma$, then we write

$$\|\boldsymbol{b}_\sigma\|_{\dot{\Lambda}_{\beta_\sigma}} = \|b_{\sigma(1)}\|_{\dot{\Lambda}_{\beta_{\sigma(1)}}} \cdots \|b_{\sigma(i)}\|_{\dot{\Lambda}_{\beta_{\sigma(i)}}}.$$

For the product of all the functions, we simply write

$$\|\boldsymbol{b}\|_{\dot{\Lambda}_\beta} = \prod_{i=1}^m \|b_i\|_{\dot{\Lambda}_{\beta_i}},$$

where $\sum\limits_{i=1}^m \beta_i = \beta$.

Definition 1.1 *Let $\varepsilon > 0$ and ψ be a fixed function which satisfies the following properties*:

(1) $\int \psi(x)\mathrm{d}x = 0$;

(2) $|\psi(x)| \le C(1 + |x|)^{-(n+\varepsilon)}$;

(3) $|\psi(x + y) - \psi(x)| \le C|y|^\varepsilon(1 + |x|)^{-(n+1+\varepsilon)}$, $2|y| < |x|$.

The multilinear commutators of Littlewood-Paley operator is defined by

$$g_{\psi,\boldsymbol{b}}(f)(x) = \Big(\int_0^\infty |F_{\boldsymbol{b},t}(f)(x)|^2 \frac{\mathrm{d}t}{t}\Big)^{\frac{1}{2}}, \tag{1.1}$$

where

$$F_{\boldsymbol{b},t}(f)(x) = \int_{\mathbf{R}^n} \prod_{i=1}^m (b_i(x) - b_i(y))\psi_t(x - y)f(y)\mathrm{d}y,$$