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Bayesian Estimation for the Order of INAR(q) Model

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Abstract: In this paper, we consider the problem of determining the order of INAR(q) model on the basis of the Bayesian estimation theory. The Bayesian estimator for the order is given with respect to a squared-error loss function. The consistency of the estimator is discussed. The results of a simulation study for the estimation method are presented.

Key words: INAR(q) model, Bayesian estimation, squared-error loss function, consistency

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1 Introduction

The time series models are widely used in many fields and there is a growing interest in time series. Estimating the order of these models is an important part when dealing the data. Chen[1] gave the Bayesian estimator for the orders of AR models with a squared-error loss function. Wang[2] discussed the problem of determining the orders of AR(k) and ARMA(p, q) models of time series on the basis of the Bayesian estimate theory. Peng[3] took advantage of the Bayesian factor to discuss the problem of selecting the order of AR models. Peng[4] gave the Bayesian estimation of the order for MA model.

The first INAR(1) model was introduced by Al-Osh and Alzaid[5]. The INAR(1) model is as follows:

$$X_t = \alpha \circ X_{t-1} + \epsilon_t, \qquad \alpha \in [0, 1],$$

where $\{\epsilon_t\}$ is an i.i.d. Poisson random sequence having mean λ; $\alpha \circ X_t = \sum\limits_{j=1}^{X_t} B_j$, where B_j an i.i.d. Bernoulli random sequence with $P\{B_j = 1\} = \alpha$ and is independent of ϵ_t. Al-Osh and Alzaid[5] gave several methods for estimating the parameters of the model. Until now, the estimation for the order of integer-valued model has not been discussed.

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Define the INAR(q) process $\{Y_t\}$ by

$$Y_t + \beta_1 \circ Y_{t-1} + \cdots + \beta_q \circ Y_{t-q} = \epsilon_t, \qquad \beta_i \in [0, 1], \quad i = 1, \cdots, q, \tag{1.1}$$

where $\{\epsilon_t\}$ is an i.i.d. Poisson random sequence having mean λ_q; $\beta_0 = 1$, $\beta_{q+l} = 0$, $l = 1, 2, \cdots$; $\beta_i \circ Y_{t-i} = \sum\limits_{j=1}^{Y_{t-i}} B_j$ for $i = 1, 2, \cdots, q$, where B_j an i.i.d. Bernoulli random sequence with $P\{B_j = 1\} = \beta_i$ and is independent of ϵ_{t-i}; $\beta(x) = 1 + \beta_1 x + \cdots + \beta_q x^q \neq 0$, $|x| \leq 1$; the order of this model is a discrete type random variable such that

$$P\{q = i\} = P_i > 0, \quad i = 1, 2, \cdots, M, \qquad \sum_{i=1}^{M} P_i = 1,$$

where M is the upper bound.

The INAR(q) process can be expressed as

$$\beta(B) \circ Y_t = \epsilon_t,$$

where $\beta(B) = 1 + \beta_1 B + \cdots + \beta_q B^q$, $\beta_i B^i \circ Y_t$ is defined by

$$\beta_i B^i \circ Y_t = \beta_i \circ Y_{t-i}, \qquad i = 1, \cdots, q.$$

Based on Theorem 2.2.2 in [6], the INAR(q) model defined above is also equivalent to the following model:

$$Y_t = \sum_{j=0}^{\infty} \alpha_j \circ \epsilon_{t-j}, \qquad \alpha_0 = 1,$$

where $|\alpha_j| \leq c\rho^{-j}$, $j \geq 0$, c and ρ are positive constants. Furthermore, we have

$$\sum_{j=0}^{\infty} |\alpha_j| < \infty, \qquad \sum_{j=0}^{\infty} |\alpha_j|^2 < \infty.$$

The INAR(q) model above can be expressed as

$$Y_t = \alpha(B) \circ \epsilon_t,$$

where

$$\alpha(B) = 1 + \alpha_1 B + \cdots + \alpha_n B^n + \cdots, \qquad \beta(B) = \frac{1}{\alpha(B)}.$$

The paper provides the Bayesian approach for the order of INAR(q) model under the assumption that the loss function is the squared-error loss function and the order is a discrete type random variable and has an upper bound. In Section 2, the relationship of the parameters of the INAR(q) model is discussed. The Bayesian estimator for the order is given and its strong consistency is discussed in Section 3. Finally, in Section 4, some simulation results are given to prove the consistency.

2 Property of INAR(q)

The mean of the model $\{Y_t\}$ is simply

$$\mathrm{E}Y_t = \frac{\lambda_q}{1 + \beta_1 + \cdots + \beta_q}.$$

Based on [7],

$$\mathrm{Cov}(Y_t, \beta_i \circ Y_{t-i}) = \beta_i \mathrm{Cov}(Y_t, Y_{t-i}) + f_i(\beta_1, \cdots, \beta_q)\mathrm{E}Y_t, \qquad i = 1, 2, \cdots, q,$$