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On the Adomian Decomposition Method for Solving PDEs

ZHU SONG-PING[1,2] AND LEE JONU[2]

(*1. School of Mathematics, Jilin University, Changchun, 130012*)

(*2. School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, NSW 2522, Australia*)

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Abstract: In this paper, we explore some issues related to adopting the Adomian decomposition method (ADM) to solve partial differential equations (PDEs), particularly linear diffusion equations. Through a proposition, we show that extending the ADM from ODEs to PDEs poses some strong requirements on the initial and boundary conditions, which quite often are violated for problems encountered in engineering, physics and applied mathematics. We then propose a modified approach, based on combining the ADM with the Fourier series decomposition, to provide solutions for those problems when these conditions are not met. In passing, we shall also present an argument that would address a long-term standing "pitfall" of the original ADM and make this powerful approach much more rigorous in its setup. Numerical examples are provided to show that our modified approach can be used to solve any linear diffusion equation (homogeneous or non-homogeneous), with reasonable smoothness of the initial and boundary data.

Key words: Adomian decomposition method, non-smooth initial condition, linear PDEs

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1 Introduction

The Adomian decomposition method (ADM), as a powerful numerical approach initially proposed by Adomian[1]–[4] in 1980s, has been successfully applied to solve various ordinary differential equations (ODEs). And yet, its extension to solving partial differential equations

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E-mail address: spz@uow.edu.au (Zhu S P).

(PDEs) are far less investigated and many issues remain to be resolved before the powerfulness of this method can be further stretched into the territory of PDEs. In this paper, we discuss the applicability of the ADM to the simplest PDE, the linear parabolic PDEs with inhomogeneous terms and demonstrate, through a proposition, that one needs to be careful in extending ADM to PDEs as there is a very strong requirement of the smoothness of the boundary and initial data. On the other hand, quite a lot of problems frequently encountered in engineering and applied mathematics do not have sufficient smoothness in terms of boundary and/or initial conditions and a direct imposition of the ADM, as some authors did, would lead to a wrong solution. For example, the solutions of Example 4 in [5] and Examples 1 and 2 in [6] satisfy the given PDEs and initial conditions but they do not satisfy the given boundary conditions. To rectify the problem, we have proposed a modified approach, which combines the ADM and Fourier expansion and leads to a much robust approach to solve linear PDEs.

Using ADM to solved ODEs has been extensively documented in the literature. The method was initially proposed by Adomian[1]–[4] and then later extended to solve various ODEs, ranging from single ones (see [7]) to systems of ODEs (see [8]), from ODEs with variable coefficients (see [9]) to highly nonlinear ones (see [10]–[11]), and from deterministic ODEs (see [12]) to stochastic equations (see [13]–[14]) or even nonlinear ODEs that exhibit singular behavior at the origin (see [9]). One of the main advantages of the ADM is that the convergence of the Adomian series solution has been proven in the context of its applications in solving ODEs (see [15]–[20]).

On the other hand, the ADM has also been applied to solve various PDEs. For example, PDEs with Dirichlet boundary conditions were studied in [21]–[23], and those with Neumann boundary conditions were studied in [12]. In terms of PDEs with very strong application backgrounds, Tatari *et al.*[24] applied the ADM to solve the Fokker-Planck equation, which is frequently used in mathematical finance, while Inc and Cherruault[25] applied it to the KdV (Korteweg-de Vries) equation, which appears frequently in mathematical physics and Arafa and Rida[26] applied it to solve the nonlinear Schrödinger equation, which often governs soliton propagation in optical fibers. In most of the papers showing ADM being applied to solve PDEs with both initial and boundary conditions (e.g., [5]–[6], [27]–[28]), all given boundary and initial conditions are composed of smooth functions. For example, using smooth boundary conditions, Wazwaz[29] and Aly *et al.*[12] suggested some strategies for finding exact solutions of nonlinear PDEs. There are also researchers who have applied ADM to coupled nonlinear PDEs (see [30]–[31]) with smooth initial conditions. Unfortunately, until now, there has been no attempt to get approximate or analytic solutions of PDEs, using the ADM, with non-smooth boundary or initial conditions.

Despite some limited attempts to extend the ADM to solve PDEs, the application of ADM to solve PDEs are relatively rare in comparison with their ODE counterparts and one of the main reasons is the lack of theoretical studies in terms of guaranteeing the solution to satisfy all the initial and boundary conditions. This is because when ADM is extended from ODEs to PDEs, its characteristic of repeatedly integrating in one direction remains, which