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Research on Ruin Probability of Risk Model Based on AR(1) Time Series

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Abstract. The insurance industry typically exploits ruin theory on collected data to gain more profits. However, state-of-art approaches fail to consider the dependency of the intensity of claim numbers, resulting in the loss of accuracy. In this work, we establish a new risk model based on traditional AR(1) time series, and propose a fine-gained insurance model which has a dependent data structure. We leverage Newton iteration method to figure out the adjustment coefficient and evaluate the exponential upper bound of the ruin probability. We claim that our model significantly improves the precision of insurance model and explores an interesting direction for future research.

AMS subject classifications: 91G05

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1 Introduction

The ruin probability was born in the last century, dating back to the doctoral dissertation which was proposed by Filip Lundberg [12] in 1903. Basically, his work laid the foundation of modern random model towards non-life fields, and he showed that the traditional random model cannot meet strict standard in mathematics. As a result, statisticians in financial industry, including many stake holders have been working on figuring out a more specific and precise model. With the guidance of classical risk models, researchers have achieved many precious results, from the theory made by Lundberg [12] to Cramér [5], who applied strict mathematical tool to figure out the initial value of $\theta(0)$, which has been proved to be $\phi(0)=\frac{1}{1+\theta}$. Also, the inequality $\phi(u)\leq e^{-Ru}$ which corresponds to the Lundberg inequality and Lundberg-Cramér equation is one of the achievements. R is the adjustment coefficient and $\phi(u)$ is the ruin probability.

Unfortunately, classical models are far too complex, and unsuitable for flexible and changeable situations in the current world. Wang *et al.* [15] did research on exponential upper bound of ruin probability when premiums depend on claims. Fu *et al.* [7] estimated the ruin probability under the condition of renewal risk model depending on time series. Meanwhile, Bao *et al.* [2] proposed a risk model in specific field where it is a Poisson process. Ruin probability has been studied depending on time series such as AR(p) model, MA(q) model, ARMA(p,q) model. Li [10] took advantage of several models in time series (AR(p) model, MA(q) model, ARMA(p, q) model and so on), analyzing ruin probability from three main areas: collecting quantities every unit time, claim amounts and interests. However, his research did not focus on number of claims. Gao *et al.* [8] studied the ruin problems when rates of interests having an autoregressive structure. Chen *et al.* [3] got the asymptotic property of ruin probability with interests where claims are pairwise negative dependent, and the interval of claims are i.i.d. Wang *et al.* [16] obtained the asymptotic estimates of the finite-time ruin probability of a dependent risk model with a constant interest rate by using the probability limiting theory and stochastic process. There are also other approaches study the dependent structure of the risk model [1,6,13,14,17] and the asymptotic property of the ruin probability [4,9,11]. Previous research of AR(p) and MA(q) models mainly focused on the dependent of interest rate, our model focus on the number of claims.

In the classical model, the number of claims, denoted as N, is an integer. The quantitative analysis of current insurance industry is based on the assumption that the claims form a sequence of independent, and identically distributed random variables. The shortcoming of existing model lies in the fact that these claims are