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HYPERCYCLIC MULTIPLICATION COMPOSITION OPERATORS ON WEIGHTED BANACH SPACE[*†]

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Abstract

This paper characterizes some sufficient and necessary conditions for the hypercyclicity of multiples of composition operators on $H^\infty_{\log,0}$.

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1 Introduction

Let $\mathbb{D}$ be an open unit disk in the complex plane $\mathbb{C}$. $H(\mathbb{D})$ denotes the space of all holomorphic functions on $\mathbb{D}$. $S(\mathbb{D})$ is the class of all holomorphic functions from the the open unit disk $\mathbb{D}$ in itself. Throughout this paper, log denotes the natural logarithm function, and ν denotes what we call a weight on $\mathbb{D}$; that is, ν is a bounded, continuous and strictly positive function defined on $\mathbb{D}$. The weighted Banach spaces of holomorphic functions $H^\infty_\nu(\mathbb{D})$, H^∞_ν for short, consists of all $f \in H(\mathbb{D})$ such that

$$\|f\|_\nu = \sup_{z\in\mathbb{D}} \nu(z)|f(z)| < \infty.$$

$H^\infty_{\nu,0}$ is the subspace of H^∞_ν consisting of $f \in H^\infty_\nu$ for which

$$\lim_{|z|\to 1} \nu(z)|f(z)| = 0.$$

Endowed with the weighted sup-norm $\|\cdot\|_\nu$, H^∞_ν and $H^\infty_{\nu,0}$ are both Banach spaces. As we all know the set of polynomials is dense in $H^\infty_{\nu,0}$, so that $H^\infty_{\nu,0}$ is a separable

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space. Particularly, let $\nu(z) = (1 - \log(1 - |z|^2))^\alpha$, $\alpha < 0$, and we will state and prove our main results acting on $H^\infty_{\log,0}$.

Each $\varphi \in S(\mathbb{D})$ induces a linear composition operator $C_\varphi : H(\mathbb{D}) \to H(\mathbb{D})$ defined by $C_\varphi f(z) = f(\varphi(z))$ with $f \in H(\mathbb{D})$ and $z \in \mathbb{D}$. A steadily increasing amount of attention has been paid to composition operator. We refer the reader to [3, 11, 12] for more details.

Let $\mathcal{L}(X)$ denote the space of linear and continuous operators on a separable, infinite dimensional Banach space X. Let $T \in \mathcal{L}(X)$. T is said to be hypercyclic if there is a vector $x \in X$ such that the orbit, defined as

$$Orb(T, x) := \{x, Tx, T^2x, T^3x, \cdots\},$$

is dense in X. Such a vector x is said to be a *hypercyclic vector* for the operator T.

The first example of a hypercyclic operator on a Banach space was given by Rolewicz [10] in 1969, which showed that if B is the unweighted unilateral backward shift on l^2, then λB is hypercyclic if and only if $|\lambda| > 1$. The notion of hypercyclic was first introduced into the field of linear dynamics in Kitai's doctoral dissertation [5], and since then this notion has been studied actively; see [1,4,6,7], and the references therein.

There are several forms to test the hypercyclicity of an operator, for examples [5, 9]. The following criterion is due to [4].

Theorem 1.1(Hypercyclicity Criterion) *Let T be an operator on a separable Banach space X. If there are dense subsets X_0, Y_0 in X, an increasing sequence $\{n_k\}$ of positive integers, and a map $S : X_0 \to X_0$ satisfying*

(i) $T^{n_k}x \to 0$ *for all* $x \in X_0$ *as* $k \to \infty$*;*

(ii) $S_{n_k}x \to 0$ *and* $T^{n_k}S_{n_k}y \to y$ *as* $k \to \infty$*, for all* $y \in Y_0$*.*

Then T is hypercyclic.

Let us recall a few preliminary facts and definitions on linear fractional transformations from [11]. We denote by $LFT(\widehat{\mathbb{C}})$ the group of linear fractional transformations, consisting of those bijections of the extended complex plane $(\widehat{\mathbb{C}}) = \mathbb{C} \cup \{\infty\}$ that are of the form $\varphi(z) = \frac{az+b}{cz+d}$, where $ad - bc \neq 0$. Two elements φ, ψ in $LFT(\widehat{\mathbb{C}})$ are said to be conjugate provided $\varphi = \sigma^{-1} \circ \psi \circ \sigma$ for some $\sigma \in LFT(\widehat{\mathbb{C}})$. The linear fractional composition operators are induced by members of the class $LFT(\mathbb{D}) = \{\varphi \in LFT(\widehat{\mathbb{C}}) : \varphi(\mathbb{D}) \subseteq \mathbb{D}\}$, and the invertible ones are induced by members of the subclass $Aut(\mathbb{D}) = \{\varphi \in LFT(\widehat{\mathbb{C}}) : \varphi(\mathbb{D}) = \mathbb{D}\}$ of automorphisms on the unit disc. The elements of $LFT(\mathbb{D})$ have the following fixed point configuration:

(a) Maps with interior fixed point. By the Schwarz lemma the interior fixed point is either attractive, or the map is an elliptic automorphism.

(b) Parabolic maps. Its fixed point is on $\partial\mathbb{D}$, and the derivative is 1 at the fixed