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Solution Remapping Technique to Accelerate Flow Convergence for Finite Volume Methods Applied to Shape Optimization Design

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Abstract. A solution remapping technique is applied to transonic airfoil optimization design to provide a fast flow steady state convergence of intermediate shapes for the finite volume schemes in solving the compressible Euler equations. Specifically, once the flow solution for the current shape is obtained, the flow state for the next shape is initialized by remapping the current solution with consideration of mesh deformation. Based on this strategy, the formula of deploying the initial value for the next shape is theoretically derived under the assumption of small mesh deformation. Numerical experiments show that the present technique of initial value deployment can attractively accelerate flow convergence of intermediate shapes and reduce computational time up to 70% in the optimization process.

AMS subject classifications: 49Q10, 65M08, 65M22
Key words: Solution remapping technique, airfoil shape optimization, finite volume scheme, initial value.

1. Introduction

The high computational cost of flow simulations is a great challenge in aerodynamic shape optimization based on computational fluid dynamics (CFD), because the optimization process often contains a large number of intermediate shapes, and the flow of each intermediate shape should be computed with the CFD solver usually to the steady state to evaluate its aerodynamic performance. For most CFD solvers, the flow governing equations are solved with some iterative scheme from a prespecified initial state, and the choice of the initial value largely influences the flow convergence.

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Theoretically, a better initial value can lead to faster convergence [2] and thus less computational cost. Therefore, a proper initial value, better than the common one based on the freestream flow, is important to raise the efficiency of the CFD solver in the optimization process.

There are two popular approaches in the deployment of initial flow state for the intermediate shapes. One is to use the freestream flow state. This method can always guarantee flow convergence, but the computational efficiency is low in the optimization process, because the time cost of each intermediate shape is almost the same as that for the first shape. The other is to copy the converged flow solution of the current intermediate shape to the next one [14, 24, 25]. This approach works under the assumption that the two shapes are very close. It is found that this method may get slower convergence in comparison to the former if the shape change is slightly larger. The possible reason is that the mesh deformation brought by the change of the shape is not taken into consideration. We believe that if the mesh deformation is involved in the construction of the initial value, the convergence rate may be further increased even for larger shape change.

In this work, we present a solution remapping technique for the initial value deployment for the finite volume methods [2, 8, 9], which are the dominant CFD solvers in aerodynamic optimization design [1,11,15]. To take into consideration the influence of mesh deformation, we remap the solution of the current shape to the next one by assuming that the mesh for the next shape is deformed from the mesh of the current shape. Then we derive the formula of deploying the initial state on mesh of the next shape. The present approach can further improve the convergence rate of the finite volume solver. Numerical experiments have shown that this method can significantly reduce the time cost in a two-dimensional transonic airfoil design problem and brings nearly no difference to the final design.

The rest of the paper is organized as follows. The transonic airfoil design problem is formulated in Section 2. Numerical methods employed in this optimization problem, including the solution remapping technique, are described in Section 3. Numerical results are reported in Section 4. Concluding remarks are given in Section 5.

2. Problem formulation

The two-dimensional transonic airfoil design problem considered in this paper is defined as follows [23]: Minimize

$$F(\mathbf{X}) = C_d \tag{2.1}$$

subject to

$$g_1(\mathbf{X}) = 1 - \frac{C_l}{C_{l0}} \leq 0, \quad g_2(\mathbf{X}) = 1 - \frac{A}{A_0} \leq 0, \tag{2.2}$$

where $\mathbf{X}$ is the vector of design variables that represents the airfoil shape, C_l and C_d are, respectively, the aerodynamic lift and drag coefficients, A is the profile area of the