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Meshfree Finite Volume Element Method for Constrained Optimal Control Problem Governed by Random Convection Diffusion Equations

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Abstract. In this paper, we investigate a stochastic meshfree finite volume element method for an optimal control problem governed by the convection diffusion equations with random coefficients. There are two contributions of this paper. Firstly, we establish a scheme to approximate the optimality system by using the finite volume element method in the physical space and the meshfree method in the probability space, which is competitive for high-dimensional random inputs. Secondly, the a priori error estimates are derived for the state, the co-state and the control variables. Some numerical tests are carried out to confirm the theoretical results and demonstrate the efficiency of the proposed method.

AMS subject classifications: 49J20, 65N35

Key words: Optimal control problem, stochastic convection diffusion equations, meshfree method, radial basis functions, finite volume element.

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1 Introduction

Optimal control problems governed by partial differential equations (PDEs) are crucial to many real-life applications, and have become an important research field in applied science. Numerical methods for PDEs have been a major research topic in applied mathematics and control theory. Much research has been carried out for these optimal control problems on theoretical analysis and numerical computation, see such as [12, 15–19, 27, 29, 30, 33].

In many complex physical and engineering models, uncertainties arise from various sources such as the coefficients, boundary conditions, forcing term or parameter of the partial differential equations. These models are often represented by stochastic partial different equations (SPDEs). The quantities in these models are usually provided by measurement data, which will lead to errors and impact the computational efficiency of the optimal control problems. So it is important to take into account the uncertainties for optimal control problems governed by SPDEs.

Although theoretical analysis and numerical approximation have been investigated for many years in dealing with the deterministic optimal control problems governed by PDEs, optimal control problems governed by SPDEs with random coefficient have become popular research field only in recent years. There exist many methods for solving these stochastic optimal control problem, such as Monte Carlo method, stochastic Galerkin method, stochastic collocation method and so on. The Monte Carlo (MC) method is one of the most commonly used methods for dealing with simulating elliptic SPDEs. The MC method is very robust and can deal with SPDEs of high dimensions, and has been used to solve optimal control problem governed by SPDEs in [5]. However, the convergence of MC method is slow for the SPDEs as reported and verified in [2]. The stochastic Galerkin method is another method which has been widely used to solve stochastic problems by using the Wiener-Hermite polynomial chaos expansion. In [13, 24–26, 28, 31], the stochastic Galerkin method has been applied to solve a series of optimal control problems governed by SPDEs. However, the stochastic Galerkin method could produce a fully coupled system of linear equations for the SPDEs and this would bring difficulty for solving the problems of high dimensions. The stochastic collocation is another efficient method for SPDEs, which leads to uncoupled linear systems and could be solved parallelly. In [9–11, 32], the stochastic collocation method is used to solve the optimal control problems with SPDEs constraints.

Since it is difficult to give the mesh partition in a high-dimensional space, the meshfree method has been developed. There are several meshfree methods