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Advantage and Disadvantage of Dispersal in Two-Species Competition Models

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Abstract. We consider a two-species competition model in which both populations are identical except their movement strategies: One species moves upward along the fitness gradient, while the other does not diffuse. While both species can coexist in homogeneous environment, we show that the species with directed movement has some advantage over the non-diffusing species in certain measurement. In contrast, if one species moves by random dispersal while the other does not diffuse, then the non-diffusing population could have advantage. Understanding the full dynamics of these ODE-PDE hybrid systems poses challenging mathematical questions.

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1 Introduction

An important question in spatial ecology is whether dispersal could convey some ecological or evolutionary advantages [1, 7, 17]. While random dispersal is selected against in spatially varying but temporally constant environment [9, 11, 16], there are plenty of examples which illustrate that dispersal can be favored [3, 13, 14]. For instance, it was shown in [18] that for a two-species competition model in which both species are subject to random dispersal and passive drift, the species with the larger dispersal rate could always outcompete the species with the smaller dispersal rate, resulting the evolution of faster dispersal; See also [15, 21, 22] for further developments.

The main aim of this paper is to consider a deceptively simply looking scenario, in which two competing populations are identical in their competitive strengths but differ in their dispersal strategies in the following way: One species adopts certain pattern of

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dispersal but the other simply does not move. We ask: Will dispersal be advantageous or disadvantageous? To this end, we first consider

$$\begin{cases} u_t = -\nabla\cdot\Big(u\nabla(m-u-v)\Big)+u(m-u-v), & x\in\Omega,\ t>0,\\ v_t = v(m-u-v), & x\in\Omega,\ t>0,\\ u\cdot\frac{\partial(m-u-v)}{\partial\nu}=0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x),\quad v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

in a bounded domain $\Omega\subset\mathbb{R}^n$ with smooth boundary $\partial\Omega$. Here $u(x,t)$ and $v(x,t)$ are densities of two populations at location x and time t. ν denotes the outward unit normal vector on $\partial\Omega$, and the boundary condition in (1.1) means that there is no individuals of the species u across $\partial\Omega$.

We envision m as the available resource distribution in the habitat, and that $m-u-v$ is the effective growth rate for both populations. In fact, $m-u-v$ is often termed as the fitness of populations. In (1.1), the term $\nabla(m-u-v)$ suggests that the movement of species u is solely governed by the advection upward along the fitness gradient, referred as fitness-dependent dispersal henceforth, so that the species u can potentially find locations with the highest fitness in order to gain some competitive advantage. In recent years there has been increasing interest in understanding the effect of fitness-dependent dispersal on population dynamics: The single species model, i.e. (1.1) with $v\equiv 0$, was proposed by Cosner [6] and its dynamics has been studied in [8]; See also [4]. For two-species competition model with a combination of both random and fitness-dependent movement, we refer to [5,20]; See also [2,12] for other types of reaction-diffusion models with fitness-dependent dispersal.

In this paper we seek for rigorous evidence for the conjecture that the dispersive strategy in (1.1) provides the respective subpopulation with an evolutionary advantage. To see why this could be a difficult task, assume that m is strictly positive in $\bar\Omega$. Then (1.1) has a continuum of positive steady states given by $\{(u^*,m-u^*)\}$, where u^* satisfies $0<u^*(x)<m(x)$ in $\bar\Omega$. This implies that both populations in (1.1) could coexist. Hence, dispersal does not seem to affect the persistence of populations in this model. However, in some biological situations the total biomass of populations could also be important, e.g. populations with small size could be subject to greater danger of extinction due to stochastic effects. Therefore, a natural question will be: How does dispersal affect the relative population size, as an index of measurement of the advantage, in two competing species model (1.1)? Accordingly, throughout this paper we focus on the case

$$m\equiv const. \qquad \text{in } \Omega. \tag{1.2}$$

We recall the following existence result from [20, Theorem 1.2]:

Theorem 1.1. *Suppose that $n\geq 1$ and that $\Omega\subset\mathbb{R}^n$ is a bounded convex domain with smooth boundary, and $m\equiv const$. Then for any choice of nonnegative functions $u_0\in\bigcup_{\gamma\in(0,1)}C^\gamma(\bar\Omega)$ and*