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An Adaptive Nonmonotone Projected Barzilai-Borwein Gradient Method with Active Set Prediction for Nonnegative Matrix Factorization

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Abstract. In this paper, we first present an adaptive nonmonotone term to improve the efficiency of nonmonotone line search, and then an active set identification technique is suggested to get more efficient descent direction such that it improves the local convergence behavior of algorithm and decreases the computation cost. By means of the adaptive nonmonotone line search and the active set identification technique, we put forward a global convergent gradient-based method to solve the nonnegative matrix factorization (NMF) based on the alternating nonnegative least squares framework, in which we introduce a modified Barzilai-Borwein (BB) step size. The new modified BB step size and the larger step size strategy are exploited to accelerate convergence. Finally, the results of extensive numerical experiments using both synthetic and image datasets show that our proposed method is efficient in terms of computational speed.

AMS subject classifications: 15A23, 65F30

Key words: Active set, projected Barzilai-Borwein method, adaptive nonmonotone line search, modified Barzilai-Borwein step size, larger step size.

1. Introduction

Nonnegative matrix factorization [11, 17, 18, 22, 23] is a classical linear dimensionality reduction method for nonnegative data, which has become an increasingly important tool for high-dimension data analysis. In general, the basic NMF problem can be stated as follows: given an $m \times n$ data matrix $V = (V_{ij})$ with $V_{ij} \geq 0$ and a pre-specified positive integer $r < \min(m, n)$, NMF intends to find two nonnegative matrices $W \in \mathbb{R}_+^{m\times r}$ and $H \in \mathbb{R}_+^{r\times n}$ such that

$$V \approx WH, \tag{1.1}$$

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where $\mathbb{R}_+^{m\times n}$ denotes $m \times n$ nonnegative matrix.

A widely used model for solving NMF (1.1) is

$$\begin{cases} \min\limits_{W,H} f(W,H) \equiv \dfrac{1}{2}\|V - WH\|_F^2 \\ \quad \text{subject to} \;\; W \geq 0, H \geq 0, \end{cases} \tag{1.2}$$

where $\|\cdot\|_F$ is the Frobenius norm. Since the role of W and H are perfectly symmetric, in this paper, we mainly consider the update of the matrix W using the modified projected Barzilai-Borwein gradient method. Let H^k denote the approximate value of H after the kth update, and let

$$f(W,H^k) = \frac{1}{2}\|V - WH^k\|_F^2, \qquad \forall k. \tag{1.3}$$

One popular and powerful method for solving (1.3) is the projected Barzilai-Borwein (PBB) gradient method, which was originated by Barzilai and Borwein [3]. At present, many papers [8, 24, 25, 27, 28] have found that PBB gradient method is an efficient method for optimization problems. Due to its simplicity and numerical efficiency, the PBB gradient method has also been successfully extended to NMF (see [4, 14, 19, 20, 31]).

Recently, Huang et al. [15] proposed a quadratic regularization nonmonotone PBB gradient method and established the global convergence result. Very recently, they discussed the monotone PBB gradient method and showed its global convergence, and the numerical experiments in [16] illustrate that the monotone PBB gradient method is better than the nonmonotone one in some cases. However, these PBB gradient methods often suffer from slow convergence due to the inexact approximations of the Hessian. It motivates us to develop new faster method for NMF.

In this paper, in order to accelerate the convergence rate and decrease the computation cost, we first use $\nabla f(W)$ to build a prox-linear approximation of $f(W,H^k)$ at W^k. We then propose a global convergent gradient-based method to solve the prox-linear approximation problem based on the alternating nonnegative least squares framework, in which we present an adaptive nonmonotone term to improve the efficiency of nonmonotone line search rule in [1] and improve the active set identifying technique in [6] such that it is reasonable to identify active constraints for NMF. To accelerate the algorithm, the modified new BB step size and the larger step size strategy are adopted. Finally, numerical experiments on synthetic and image datasets illustrate the efficiency of our method.

This paper is organized as follows. In Section 2, we propose an efficient method for NMF and discuss the global convergence of the proposed method in Section 3. The experimental results are presented in Section 4. Finally, Section 5 concludes the work. Throughout the paper, the symbol $\|\cdot\|_F$ denotes the Frobenius norm of matrices, $\langle\cdot,\cdot\rangle$ denotes the inner product of two matrices.