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Cotorsion Dimension of Weak Crossed Products

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Abstract: Let H be a finite-dimensional weak Hopf algebra over a field k and A an associative algebra, and $A\#_\sigma H$ a weak crossed product. In this paper, a spectral sequence for Ext is constructed which yields an estimate for cotorsion dimension of $A\#_\sigma H$ in terms of the corresponding data for H and A.

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1 Introduction

In 1996, Böhm and Szlachányi[1] introduced weak bialgebras (or weak Hopf algebras) as a generalization of ordinary bialgebras (or Hopf algebras). A general theory for these objects was subsequently developed in Böhm *et al.*[2]. Briefly, the axioms of a weak Hopf algebra are the same as the ones for a Hopf algebra, except that the coproduct of the unit, the product of the counit and the antipode conditions are replaced by weaker properties. The main motivation for studying weak Hopf algebras comes from quantum field theory, operator algebras and representation theory. It has turned out that many results of classical Hopf algebra theory can be generalized to weak Hopf algebras. Shen[3] extended the theory of crossed products were introduced independently by Blattner and Montgomery[4], Doi and Takeuchi[5] to more general Hopf structure: weak Hopf algebras. At the categorical level, Alonso Álvarez and González Rodríguez[6] introduced the notion of a weak crossed product and Alonso Álvarez *et al.*[7] investigated weak cleft theory and weak Galois extensions for weak Hopf algebras (see [8] and [9]).

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In 2005, Mao and Ding[10] introduced the cotorsion dimension of modules and rings. Recently, Chen *et al.*[11] discussed the cotorsion dimension of the smash product $A\#H$, which generalizes the result of group rings introduced by Bennis and Mahdou[12]. It is now very natural to ask whether cotorsion dimension of the weak crossed products, the weak crossed products we consider here are generalizations of the crossed products and weak smash products. This question motivates the present research.

This paper is organized as follows: In Section 2, we recall some basic definitions and results such as cotorsion dimension, weak Hopf algebras, weak crossed products and so on. In Section 3, we mainly investigate the relationship between the global cotorsion dimension of the weak crossed product $A\#_\sigma H$ with the algebra A.

2 Preliminaries

Throughout this paper, we work over a commutative field k. All algebras, linear spaces etc. are over k; unadorned $\otimes$ means $\otimes_k$.

2.1 Cotorsion Dimension

The cotorsion dimension of an A-module M denoted by $cd_A(M)$ is the least positive integer n satisfying $\mathrm{Ext}_A^{n+1}(F,\ M) = 0$ for all flat A-modules F. In particular, if $cd_A(M) = 0$, then M is called cotorsion. The right global dimension of A is denoted by $\mathrm{r}.D(A)$. The left global cotorsion dimension of A, denoted by $\mathrm{l.cot}.D(A)$, is defined as the supremum of the cotorsion dimensions of A-modules (see [10]).

2.2 Weak Hopf Algebras

For the basic definitions and properties of weak Hopf algebras, see [2]. Recall that a weak Hopf algebra H is an algebra $(H,\ m,\ \mu)$ and coalgebra $(H,\ \Delta,\ \varepsilon)$ such that for $h, k, l \in H$, the following axioms hold:

(1) $\Delta(hk) = \Delta(h)\Delta(k)$;

(2) $\Delta^2(1) = 1_{(1)} \otimes 1_{(2)}1_{(1')} \otimes 1_{(2')} = 1_{(1)} \otimes 1_{(1')}1_{(2)} \otimes 1_{(2')}$;

(3) $\varepsilon(hkl) = \varepsilon(hk_{(1)})\varepsilon(k_{(2)}l) = \varepsilon(hk_{(2)})\varepsilon(k_{(1)}l)$;

(4) There exists a k-linear map $S\colon H \longrightarrow H$, called the antipode, satisfying

$$h_{(1)}S(h_{(2)}) = \varepsilon(1_{(1)}h)1_{(2)},$$
$$S(h_{(1)})h_{(2)} = 1_{(1)}\varepsilon(h1_{(2)}),$$
$$S(h) = S(h_{(1)})h_{(2)}S(h_{(3)}).$$

We have idempotent maps ε_t, ε_s, $\bar{\varepsilon}_t$, $\bar{\varepsilon}_s$: $H \longrightarrow H$ defined by

$$\varepsilon_t(h) = \varepsilon(1_{(1)}h)1_{(2)}, \qquad \varepsilon_s(h) = 1_{(1)}\varepsilon(h1_{(2)}),$$
$$\bar{\varepsilon}_t(h) = \varepsilon(h1_{(1)})1_{(2)}, \qquad \bar{\varepsilon}_s(h) = 1_{(1)}\varepsilon(1_{(2)}h),$$