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Extensions of Modules with ACC on d-annihilators

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Abstract: A unitary right R-module M_R satisfies acc on d-annihilators if for every sequence $(a_n)_n$ of elements of R the ascending chain $\mathrm{Ann}_M(a_1) \subseteq \mathrm{Ann}_M(a_1a_2) \subseteq \mathrm{Ann}_M(a_1a_2a_3) \subseteq \cdots$ of submodules of M_R stabilizes. In this paper we first investigate some triangular matrix extensions of modules with acc on d-annihilators. Then we show that under some additional conditions, the Ore extension module $M[x]_{R[x;\alpha,\delta]}$ over the Ore extension ring $R[x;\, \alpha, \delta]$ satisfies acc on d-annihilators if and only if the module M_R satisfies acc on d-annihilators. Consequently, several known results regarding modules with acc on d-annihilators are extended to a more general setting.

Key words: triangular matrix extension, Ore extension, acc on d-annihilator

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1 Introduction

Throughout this paper all rings R are associative with identity and all modules M_R are unitary right R-modules. The set of all positive integers is denoted by $\mathbf{N}_+$. Let α be an endomorphism and δ an α-derivation of a ring R. We denote by $R[x;\, \alpha, \delta]$ the Ore extension whose elements are the polynomials over R, the addition is defined as usual and the multiplication is subject to the relation $xa = \alpha(a)x + \delta(a)$ for any $a \in R$. Clearly, polynomial rings $R[x]$, skew polynomial rings $R[x;\, \alpha]$ and differential polynomial rings $R[x;\, \delta]$ are special

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Ore extension rings. Given a right R-module M_R, we can make $M[x]$ into a right $R[x;\,\alpha,\delta]$-module by allowing polynomials from $R[x;\,\alpha,\delta]$ to act on polynomials in $M[x]$ in the obvious way, and apply the above twist whenever necessary. The verification that this defines a valid $R[x;\,\alpha,\delta]$-module structure on $M[x]$ is almost identical to the verification that $R[x;\,\alpha,\delta]$ is a ring and it is straightforward (see [1]).

For an element $a \in R$, $\mathrm{Ann}_M(a) = \{m \in M_R \mid ma = 0\}$ denotes the annihilator of a in M_R. Following Frohn[2], a module M_R is said to satisfy acc on d-annihilators if for every sequence $(a_n)_n$ of elements of R, the ascending chain $\mathrm{Ann}_M(a_1) \subseteq \mathrm{Ann}_M(a_1a_2) \subseteq \cdots$ of submodules of M_R stabilizes. If R_R satisfies acc on d-annihilators, then we say that the ring R is a ring satisfying acc on d-annihilators. Clearly, strongly Laskerian modules satisfy acc on d-annihilators, and if M_R satisfies acc on d-annihilators, so is every submodule of M_R (see [2]). Visweswaran[3] showed that the zero-dimension rings with acc on d-annihilators are exactly the perfect rings. So in order to characterize the perfect rings R, it is important to consider the modules R_R with acc on d-annihilators. Hence find more examples of modules with acc on d-annihilators is meaningful in module theory. It is well known that, in the module theory literature, many surprising examples and counterexamples have been produced via the triangular matrix extensions. So in this paper we first investigate the relationship between the acc on d-annihilators property of M_R and that of the various triangular matrix extension modules over M_R, and then obtain more examples of modules with acc on d-annihilators.

Polynomial extension of modules with acc on d-annihilators was studied by Frohn. He proved in [2] that if R is reduced and satisfies acc on d-annihilators, then the polynomial ring $R[X]$ for any set X of indeterminates also has acc on d-annihilators. We generalize this result. In Section 3, we consider the acc on d-annihilators property of the Ore extension modules $M[x]_{R[x;\,\alpha,\delta]}$ over the Ore extension rings $R[x;\,\alpha,\delta]$. We show that if M_R is an (α,δ)-compatible reduced module, then the Ore extension module $M[x]_{R[x;\,\alpha,\delta]}$ satisfies acc on d-annihilators if and only if M_R satisfies acc on d-annihilators. So the Frohn's recent work (see [2], Corollary 2.4]) is extended to a more generally setting.

2 Triangular Matrix Extension Modules

Let R be a ring and M_R a right R-module. Let

$$U_n(R) = \left\{ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \mid a_{ij} \in R \right\}$$

and

$$U_n(M) = \left\{ \begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ 0 & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & m_{nn} \end{pmatrix} \mid m_{ij} \in M_R \right\}.$$