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Homotopy Analysis Method for Solving (2+1)-dimensional Navier-Stokes Equations with Perturbation Terms

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Abstract: In this paper Homotopy Analysis Method (HAM) is implemented for obtaining approximate solutions of (2+1)-dimensional Navier-Stokes equations with perturbation terms. The initial approximations are obtained using linear systems of the Navier-Stokes equations; by the iterations formula of HAM, the first approximation solutions and the second approximation solutions are successively obtained and Homotopy Perturbation Method (HPM) is also used to solve these equations; finally, approximate solutions by HAM of (2+1)-dimensional Navier-Stokes equations without perturbation terms and with perturbation terms are compared. Because of the freedom of choice the auxiliary parameter of HAM, the results demonstrate that the rapid convergence and the high accuracy of the HAM in solving Navier-Stokes equations; due to the effects of perturbation terms, the 3rd-order approximation solutions by HAM and HPM have great fluctuation.

Key words: Navier-Stokes equation, homotopy analysis method, homotopy perturbation method, perturbation term
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1 Introduction

Most of nonlinear partial differential equations do not have a precise analytical solution, by approximate methods these nonlinear equations can be solved. So far many numerical algorithms have been developed for the approximate solutions of nonlinear partial differential

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equations, such as variational iteration method (see [1]–[5]), homotopy analysis method (see [6]–[10]), differential transform method and the homotopy analysis method (see [11]), differential quadrature method, etc. (see [12] and [13]).

Homotopy analysis method (HAM) is one of the most effective methods to find approximate solution of nonlinear partial differential equations. The HAM always provides us with a family of solution expressions in the auxiliary parameter $\hbar$, by the convergence-controller parameter $\hbar$, the region and rate of each solution might be adjusted and controlled conveniently (see [14]). The HAM has been applied to solve many types of nonlinear problems, such as the nonlinear Cauchy problem of parabolic-hyperbolic type (see [15]), differential-difference equations (see [16]), nonlinear reaction-diffusion-convection problems (see [17]), nonlocal initial boundary value problem (see [18]), fractional differential equations (see [19]–[21]), Fredholm and Volterra integral equations (see [22]). The HAM has been also used to investigate the heat conduction problems (see [23]–[29]). Convergences of the homotopy method are studied (see, for example, [30]–[36]).

The objective of this article is to implement HAM for finding new traveling wave solutions of (2+1)-dimensional Navier-Stokes equations with perturbation terms. Navier-Stokes equations are the most important equations in fluid dynamics for finding the velocity and pressure functions (see [37]–[39]). Viscosity is a characteristic of a fluid, for example, air is a kind of high viscosity fluid, water and air are fluids with low viscosity. Meanwhile, the fluid movement equation is contained Navier-Stokes equations (see [40]).

2 Navier-Stokes Equations with Perturbation Terms

The Navier-Stokes equations with perturbation terms can be written in the following basic form:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial x}-\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial z^2}\right)=F_1, \tag{2.1}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial z}-\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial z^2}\right)=F_2, \tag{2.2}$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial z}=F_3, \tag{2.3}$$

where U and V is speed component in direction to x and z, respectively, P is the pressure, ρ is the fluid density, and ν is the kinematics of fluid coefficient that is positive constant, F_i $(i=1,2,3)$ are the perturbation terms. The Navier-Stokes equations are nonlinear partial differential equations and have perturbation terms, so approximate methods must be constructed.

We introduce a complex variable ξ, $\xi=x+z-ct$, where c is the speed of traveling wave.

Thus, (2.1)–(2.3) become the ordinary differential equations as follows:

$$-c\frac{\mathrm{d}U}{\mathrm{d}\xi}+U\frac{\mathrm{d}U}{\mathrm{d}\xi}+V\frac{\mathrm{d}U}{\mathrm{d}\xi}+\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}\xi}-2\nu\frac{\mathrm{d}^2 U}{\mathrm{d}\xi^2}=F_1, \tag{2.4}$$