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Spline R-Function and Applications in FEM

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Abstract. R-function is a widely used tool when considering objects obtained through the Boolean operations start from simple base primitives. However, there is square root operation in the representation. Considering that the use of splines will facilitate the calculations within the CAD system, in this paper, we propose a system of R-functions represented in spline form called Spline R-function (SR). After transforming the function ranges of two base primitives to a new coordinate system, a series of sign constraints following a specific Boolean operation are derived and the spline R-function can be formulated as a piecewise function. Representation of SR in both Bézier form and B-spline form have been given. Among which the Bézier ordinates are determined with the help of the B-net method through setting up a series of relations according to the sign constraints and properties of R-functions. The construction processes for both Boolean intersection and union operations with different smoothness are discussed in detail. Numerical experiments are conducted to show the potential of the proposed spline R-function.

AMS subject classifications: 68U07, 65D18, 65D30

Key words: R-functions, CSG, Boolean operations, B-splines, B-net method, WEB method.

1. Introduction

Recent years have witnessed a rapid development of 3D printing together with the related studies derived in topics such as modeling and analysis [1–7]. Due to the several modeling requirements of 3D printing, implicit representation forms an ideal choice for such technology. Implicit representation possesses a natural point classification (i.e., inside/outside the geometry) and enables various topologies, which is convenient to describe the slicing structures of different objects [8, 9]. These properties among others have increased the attention for using the implicit form to model geometries for 3D printing [10–12]. In the context of implicit solids and function representation, Constructive Solid Geometry (CSG) is adopted in the modeling process for objects with complex structures (e.g., [13, 14]).

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CSG is a technique used to get complex solid objects starting from simple primitives by a series of Boolean operations (union, intersection and difference) [15]. Sophisticated objects are produced by combining a relatively small number of primitives such as spheres, cones and cuboids. The primary tool used when dealing with Boolean operations in the literature is the R-function, which is employed to produce the final algebraic representation [16, 17].

V. L. Rvachev, originally introduced R-functions, or Rvachev functions, in the 1960s by combining logic, geometry and analysis under a unified framework [17, 18]. The method gives an algebraic representation for shapes obtained from the combinations of simple shapes through set-theoretic operations. Since then, the related theory, as well as the applications, have been extensively explored [19–22]. R-functions are applied in many areas including computer graphics, engineering design, analysis and optimization [23–26]. In geometric modeling, R-functions are used when constructing objects with complex structures (e.g., in [13, 16]). Moreover, these functions are adopted to define the geometry when solving boundary-value problems in computational physics [18, 27, 28].

Among different systems of R-functions, R_0-function is the most popular one regarding applications [17, 18]. In [19], higher smoothness of R-functions has been attained by developing a generalized R_0^m system. However, there are square root operations in such representations. Furthermore, addition and multiplication are not enough to construct R-function systems of polynomials that are sufficiently complete. According to [19], a sufficiently complete system do not need to use the root operation and R-functions can be constructed from piecewise polynomials without the need for the root operation. This leads to a good direction to present systems that includes complete Boolean operations and avoids the square root operations at the same time.

In the Computer-Aided Design (CAD) system, the dominant geometrical representation uses spline functions [29]. The growing interest in splines is due to their simple implementation and nice properties. Different types of splines have been introduced in the recent years with various studies for their proprties [30–33]. In this paper, a R-function system represented in spline form is proposed. The starting point is based on the definition of R-function, where specific Boolean operations have fixed signs that depend on the signs of their base primitives. Afterward, a straightforward procedure to map such fixed signs into a new coordinate system is conducted. One of the motivations for this work is to adopt such simple mapping in order to represent the R-function by splines. Moreover, the composition operation with splines is also convenient and can be handled well by the de Boor algorithm [29]. A detailed construction of spline R-function is presented for both Boolean intersection and union operations. In each case, different smoothness are illustrated to indicate the simplicity and practicality of the new mechanism. Results show that this new system of R-functions satisfies the requirements of the original R-functions. Additionally, the proposed spline R-functions prevent the ambiguity zeros inside the interior of the domain since no singularities appear in the representation.

The remainder of the paper is organized as follows. Section 2 presents the pre-