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Signed Roman (Total) Domination Numbers of Complete Bipartite Graphs and Wheels

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Abstract: A signed (res. signed total) Roman dominating function, SRDF (res. STRDF) for short, of a graph $G = (V,\ E)$ is a function $f\colon V \to \{-1,\ 1,\ 2\}$ satisfying the conditions that (i) $\sum\limits_{v\in N[v]} f(v) \geq 1$ (res. $\sum\limits_{v\in N(v)} f(v) \geq 1$) for any $v \in V$, where $N[v]$ is the closed neighborhood and $N(v)$ is the neighborhood of v, and (ii) every vertex v for which $f(v) = -1$ is adjacent to a vertex u for which $f(u) = 2$. The weight of a SRDF (res. STRDF) is the sum of its function values over all vertices. The signed (res. signed total) Roman domination number of G is the minimum weight among all signed (res. signed total) Roman dominating functions of G. In this paper, we compute the exact values of the signed (res. signed total) Roman domination numbers of complete bipartite graphs and wheels.

Key words: signed Roman domination, signed total Roman domination, complete bipartite graph, wheel

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1 Introduction

For notation and graph theory terminology, we in general follow [1]. Specifically, let G be a graph with vertex set $V(G) = V$ of order $|V| = n(G)$ and size $|E(G)| = m(G)$, and let v be a vertex in V. The open neighborhood of v is $N(v) = \{u \in V \mid uv \in E\}$ and the closed neighborhood of v is $N[v] = \{v\} \cup N(v)$. The degree of v is $d(v) = |N_G(v)|$. A stable set in

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G is a set of vertices such that there is no edge between two vertices of the set. A complete bipartite graph is a graph such that its vertices can be partitioned into two stable sets, and every vertex in each stable set has an edge with every vertex in the other. A wheel W_{n+1} is a graph composed with a cycle C_n and a single vertex c such that c is connected by an edge with every vertex of C_n.

Cockayne *et al.*[2] defined a Roman dominating function (RDF, for short) on a graph $G = (V,\ E)$ to be a function $f\colon V \to \{0,\ 1,\ 2\}$ satisfying the condition that every vertex v for which $f(v) = 0$ is adjacent to at least one vertex u for which $f(u) = 2$. Roman domination has been extensively studied in, for example, [3]–[9].

For a real-valued function $f\colon V \to R$ the weight of f is $w(f) = \sum\limits_{v\in V} f(v)$. For $S \subseteq V$ we define $f(S) = \sum\limits_{v\in S} f(v)$, and so $w(f) = f(V)$. Throughout the paper, we replace $f(N[v])$ with $f[v]$ for convenience.

Ahangar *et al.*[10] defined a signed Roman dominating function (SRDF) on a graph $G = (V,\ E)$ to be a function $f\colon V \to \{-1,\ 1,\ 2\}$ satisfying the conditions that (i) $f[v] \geq 1$ for every $v \in V$, and (ii) every vertex v for which $f(v) = -1$ is adjacent to a vertex u for which $f(u) = 2$. The signed Roman domination number, denoted by $\gamma_{sR}(G)$, is the minimum weight among all SRDFs in G, that is,

$$\gamma_{sR}(G) = \min\{w(f) \mid f \text{ is a SRDF in } G\}.$$

A SRDF of weight $\gamma_{sR}(G)$ is called a $\gamma_{sR}(G)$-function.

Volkmann[11] further induced a signed total Roman dominating function (STRDF) on a graph $G = (V,\ E)$ to be a function $f\colon V \to \{-1,\ 1,\ 2\}$ satisfying the conditions that (i) $f(N(v)) \geq 1$ for every $v \in V$, and (ii) every vertex v for which $f(v) = -1$ is adjacent to a vertex u for which $f(u) = 2$. The signed total Roman domination number, denoted by $\gamma_{stR}(G)$, is the minimum weight among all STRDFs in G, that is,

$$\gamma_{stR}(G) = \min\{w(f) \mid f \text{ is a STRDF in } G\}.$$

A STRDF of weight $\gamma_{stR}(G)$ is called a $\gamma_{stR}(G)$-function.

The exact values of signed Roman (res. signed total Roman) domination number of complete graphs, cycles and paths were given in [10] (res. in [11]). In the present paper, we compute the exact values of signed (res. signed total) Roman domination numbers of complete bipartite graphs and wheels.

2 Complete Bipartite Graphs

In [10], the signed Roman domination number of a complete graph has been determined. Now, we provide the signed Roman domination number of a complete bipartite graph as follows.

Theorem 2.1 *For any complete bipartite graph $K_{m,n}$ $(m \leq n)$,*

(1) $\gamma_{sR}(K_{1,n}) = \begin{cases} 2, & \text{if } n \text{ is even;} \\ 1, & \text{if } n \text{ is odd;} \end{cases}$