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The Relaxation Limits of the Two-Fluid Compressible Euler-Maxwell Equations

WANG Shu and ZHENG Lin*

College of Applied Science, Beijing University of Technology, Beijing 100124, China.

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Abstract. In this paper we consider the relaxation limits of the two-fluid Euler-Maxwell systems with initial layer. We construct an asymptotic expansion with initial layer functions and prove the convergence between the exact solutions and the approximate solutions.

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Key Words: Euler-Maxwell equation; relaxation limit; initial layer; asymptotic expansion.

1 Introduction

In this paper, we consider the three-dimensional two-fluid (including electrons and ions) Euler-Maxwell equations in a torus $\mathbb{T}=(\mathbb{R}/\mathbb{Z})^3$:

$$\partial_t n_\alpha+\mathrm{div}(n_\alpha u_\alpha)=0, \tag{1.1}$$

$$m_\alpha[\partial_t(n_\alpha u_\alpha)+\mathrm{div}(n_\alpha u_\alpha\otimes u_\alpha)]+\nabla p(n_\alpha)=q_\alpha n_\alpha(E+u_\alpha\times B)-\frac{m_\alpha n_\alpha u_\alpha}{\tau_\alpha}, \tag{1.2}$$

$$\varepsilon\partial_t E-\frac{1}{\mu}\nabla\times B=n_e u_e-n_i u_i, \tag{1.3}$$

$$\partial_t B+\nabla\times E=0, \tag{1.4}$$

$$\varepsilon\mathrm{div}E=n_i-n_e,\quad \mathrm{div}B=0, \tag{1.5}$$

where $\alpha=e,i$, $q_i=1$, $q_e=-1$; n_e and n_i stand for the density of the electrons and ions; u_e and u_i stand for the velocity of the electrons and ions; E and B are respectively the electric

*Corresponding author. *Email address:* zhenglin0103@emails.bjut.edu.cn (L. Zheng), wangshu@bjut.edu.cn (S. Wang)

field and magnetic field; $p=p(n_\alpha)$ is the pressure function which is sufficiently smooth and strictly increasing for $n_\alpha>0$. These variables are functions of a three-dimensional position vector $x\in\mathbb{T}$ and of the time $t>0$. In the above systems the physical parameters are the electron mass m_e and the ion mass m_i, the momentum relaxation times τ_e and τ_i, and the permittivity ε and the permeability μ.

For simplicity, we denote $m_\alpha=1$, $\varepsilon,\mu=1$ and $\tau_e=\tau_i=\tau$, then we obtain the following systems:

$$\partial_t n_\alpha+\mathrm{div}(n_\alpha u_\alpha)=0, \tag{1.6}$$

$$\partial_t(n_\alpha u_\alpha)+\mathrm{div}(n_\alpha u_\alpha\otimes u_\alpha)+\nabla p(n_\alpha)=q_\alpha n_\alpha(E+u_\alpha\times B)-\frac{n_\alpha u_\alpha}{\tau_\alpha}, \tag{1.7}$$

$$\partial_t E-\nabla\times B=n_e u_e-n_i u_i,\quad \mathrm{div}E=n_i-n_e, \tag{1.8}$$

$$\partial_t B+\nabla\times E=0,\quad \mathrm{div}B=0. \tag{1.9}$$

Furthermore, we make the time scaling by replacing t by $\frac{t}{\tau}$ and define the enthalpy function $h(n_\alpha)$ by

$$h(n_\alpha)=\int_1^{n_\alpha}\frac{p'(s)}{s}\mathrm{d}s. \tag{1.10}$$

So the system we considered is rewritten the following reduced two-fluid Euler-Maxwell systems:

$$\partial_t n_\alpha+\frac{1}{\tau}\mathrm{div}(n_\alpha u_\alpha)=0, \tag{1.11}$$

$$\partial_t u_\alpha+\frac{1}{\tau}(u_\alpha\cdot\nabla)u_\alpha+\frac{1}{\tau}\nabla h(n_\alpha)=\frac{q_\alpha(E+u_\alpha\times B)}{\tau}-\frac{u_\alpha}{\tau^2}, \tag{1.12}$$

$$\partial_t E-\frac{1}{\tau}\nabla\times B=\frac{n_e u_e-n_i u_i}{\tau},\quad \mathrm{div}E=n_i-n_e, \tag{1.13}$$

$$\partial_t B+\frac{1}{\tau}\nabla\times E=0,\quad \mathrm{div}B=0, \tag{1.14}$$

with initial data:

$$(n_\alpha,u_\alpha,E,B)|_{t=0}=(n^\tau_{\alpha,0},u^\tau_{\alpha,0},E^\tau_0,B^\tau_0). \tag{1.15}$$

The study of compressible Euler-Maxwell equations began in 2000, Chen, Jerome and Wang [1] prove the existence of global weak solutions of the simplified Euler-Maxwell equations by using the method of step by step Godunov scheme combined with compensated compactness; in 2007 and 2008, Peng and Wang [2,3] study the non relativistic limit convergence problem for compressible Euler-Maxwell equations to compressible Euler-Poisson equations and the composite limits of the quasi neutral limit and the non relativistic limit for compressible Euler-Maxwell equations; Peng, Wang and Gu [4] discuss the relaxation limit of compressible Euler-Maxwell equations and the existence of global smooth solution in 2011; in the same year, Wang, Yang and Zhao [5] research the relaxation limit of the plasma two-fluid Euler-Maxwell equations with the help of Maxwell