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Thermoelastic Interaction in an Infinite Long Hollow Cylinder with Fractional Heat Conduction Equation

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Abstract. In this work, we introduce a mathematical model for the theory of generalized thermoelasticity with fractional heat conduction equation. The presented model will be applied to an infinitely long hollow cylinder whose inner surface is traction free and subjected to a thermal and mechanical shocks, while the external surface is traction free and subjected to a constant heat flux. Some theories of thermoelasticity can extracted as limited cases from our model. Laplace transform methods are utilized to solve the problem and the inverse of the Laplace transform is done numerically using the Fourier expansion techniques. The results for the temperature, the thermal stresses and the displacement components are illustrated graphically for various values of fractional order parameter. Moreover, some particular cases of interest have also been discussed.

AMS subject classifications: 74K25, 74G60

Key words: Thermoelasticity, fractional calculus, hollow cylinder, Laplace transform, traction free, heat flux.

1 Introduction

The classical uncoupled theory of thermoelasticity predicts two phenomena which are not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves. Biot [1] formulated the theory of coupled thermoelasticity to overcome the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for

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both theories, however, are of the diffusion type, predicting infinite speeds of propagation for heat waves contrary to physical observations. Lord and Shulman [2] introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended by Dhaliwal and Sherief [3] to include the anisotropic case. In this theory, a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity.

In recent years fractional differential calculus applications have been developed in physics, chemistry as well as in engineering fields. Fractional order integrals and derivatives extend the well-known de nitions of integer-order primitives and derivatives of the ordinary differential calculus to real-order operators. Although fractional calculus has a long history, its research still stays in the realm of theory, because of lacking the proper mathematical methods and applied background. Until the last few decades, many researchers pointed out that it has many applications in various fields, such as in viscoelastic mechanics, power-law phenomenon in fluid and complex network, allometric scaling laws in biology and ecology, colored noise, electrode-electrolyte polarization, dielectric polarization, boundary layer effects in ducts, electromagnetic waves, quantitative finance, quantum evolution of complex systems, and fractional kinetics. Therefore, to find the solutions of the fractional differential equations becomes very important [4].

Fractional calculus has been used successfully to modify many existing models of physical processes. The first application of fractional derivatives was given by Abel who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. One can state that the whole theory of fractional derivatives and integrals was established in the second half of the 19th century. Caputo and Mainardi [5,6] and Caputo [7] found good agreement with experimental results when using fractional derivatives for description of viscoelastic materials and established the connection between fractional derivatives and the theory of linear viscoelasticity. The generalization of the concept of derivative and integral to a non-integer order has been subjected to several approaches and various alternative definitions of fractional derivatives [8–13]. One can refer to Podlubny [12] for a survey of applications of fractional calculus. Recently, the fractional order theory of thermoelasticity was derived by Sherief et al. [14]. Also, Youssef [19] introduced a new theory of thermoelasticity using the methodology of fractional calculus. Other some works in the subject are [15–18]).

In this work, a new model of theory of generalized thermoelasticity with one relaxation time utilizing the technique of fractional calculus theory is established. We consider an infinitely long annular cylinder whose inner surface is traction free and subjected to a thermal shock, the outer surface is also traction free but subjected to a constant heat flux and the medium parameters quiet introductory state. Laplace transforms techniques are used to get the solution in a closed form. Inversion of the Laplace transforms is carried out using a numerical approach.