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FITTING C^1 SURFACES TO SCATTERED DATA WITH $S_2^1(\Delta_{m,n}^{(2)})^*$

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Abstract

This paper presents a fast algorithm (BS2 Algorithm) for fitting C^1 surfaces to scattered data points. By using energy minimization, the bivariate spline space $S_2^1(\Delta_{m,n}^{(2)})$ is introduced to construct a C^1-continuous piecewise quadratic surface through a set of irregularly $3D$ points. Moreover, a multilevel method is also presented. Some experimental results show that the accuracy is satisfactory. Furthermore, the BS2 Algorithm is more suitable for fitting surfaces if the given data points have some measurement errors.

Mathematics subject classification: 41A15, 65D07, 65D10.
Key words: Bivariate spline, Scattered data, Surface fitting, Energy minimization, Type-2 triangulation, C^1-continuous.

1. Introduction

Fitting surface to scattered data is a fast growing research area. It deals with the problem of reconstructing an unknown function from given scattered data. The main aim of this paper is to solve the following problem:

> (Θ): Let D be a domain in the (x, y)-plane, and suppose F is a real-valued function defined on D. Suppose we are given the values $z_j = F(x_j, y_j)$ of F at some set of points (x_j, y_j) located in $D, j = 1, 2, \cdots, N$. Find a function f defined on D which reasonably approximates F.

This problem is, of course, precisely the problem of fitting a surface to given data. Naturally, it has many applications, such as terrain modeling, surface reconstruction, fluid-structure interaction, numerical solutions of partial differential equations, kernel learning, and parameter estimation, to name a few. Moreover, these applications come from such different fields as applied mathematics, computer science, geology, biology, engineering, and even business studies([28]).

C^1 surface is the simplest and most practical surfaces in scientific computation and engineering, and it has very important applications. There are lots of efficient methods for fitting surfaces such as Shepard's method, tensor product splines, multiquadratics(MQ), and finite element methods. Shepard defined a C^0-continuous interpolation function as the weighted average

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of the data([19]). Using tensor product B-splines, Lee *et al.* constructed a C^2-continuous interpolation function ([15]). Hardy's multiquadratics are among the most successful and applied methods, it constructs a C^∞-continuous interpolation function ([10]). However, the solvability of multiquadratics interpolation depends on the selection of parameters. Franke and Nielson introduced the modified quadratic Shepard's method to produce C^1-continuous interpolation ([7]), but it is sensitive to triangulation and data distribution, just like finite element methods ([1]). Compactly supported functions were presented in [28, 29], which could produce an interpolation function with arbitrary smoothness. However, it needs to solve some equations. In this paper, using energy minimization, we proposed a very fast algorithm (BS2 Algorithm) for reconstructing a C^1-continuous interpolation function from arbitrary scattered data.

Comparing with all of methods mentioned above, the BS2 Algorithm presented in this paper has some advantages, such as:

- the BS2 Algorithm doesn't need to solve any equations.
- the BS2 Algorithm produces a C^1-continuous function which degree is just 2.
- according to the different requirements, the BS2 Algorithm could construct a function f which interpolates the data exactly, or approximately fits the given data.
- the BS2 Algorithm is very simply because the basis which we used here is centrosymmetric (see Fig. 3.2).
- the surfaces produced by the BS2 Algorithm have minimum energy since the minimum energy constraint is used.

This paper is organized as follows. Section 2 reviews previous work. In Section 3, the spline space $S_2^1(\Delta_{m,n}^{(2)})$ is introduced. Surfaces' energy minimization is introduced in Section 4. The basic idea and the algorithm are given in Section 5. That serves to motivate the discussion of multilevel approximation, given in Section 6. In Section 7, the numerical results and comparison with some algorithms are illustrated. Conclusions and future works are presented in Section 8.

2. Previous Work

There are basically two approaches to handling (Θ). First, we may try to construct a function f which interpolates the data exactly, i.e., such that

$$f(x_j, y_j) = z_j, \qquad j = 1, 2, \cdots, N.$$

This approach may be desirable when the function values at the data points are known to high precision and where it is highly desirable that these values be preserved by the approximating function. The problem of interpolation of scattered data has been addressed by numerous authors ([1, 2, 8, 11, 24]). One of the earliest algorithm in this field was based on inverse distance weighting of data, namely Shepard's method ([7, 19]). Another popular approach to scattered interpolation is to define the interpolation function as a linear combination of radially symmetric basis functions (RBF). Popular choices for the basis functions include Gaussian, multiquadratics (MQ) ([10]), compactly supported functions(CSF) ([27, 29]). Another class of solutions to scattered data interpolation is due to finite element methods ([14]). Lee proposed an algorithm (BA Algorithm) for scattered data interpolation with B-splines ([15]). A recent view of methods for scattered data interpolation is given by [28].