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KINETIC FLUX VECTOR SPLITTING FOR THE EULER EQUATIONS WITH GENERAL PRESSURE LAWS *1)

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Abstract

This paper attempts to develop kinetic flux vector splitting (KFVS) for the Euler equations with general pressure laws. It is well known that the gas distribution function for the local equilibrium state plays an important role in the construction of the gas–kinetic schemes. To recover the Euler equations with a general equation of state (EOS), a new local equilibrium distribution is introduced with two parameters of temperature approximation decided uniquely by macroscopic variables. Utilizing the well-known connection that the Euler equations of motion are the moments of the Boltzmann equation whenever the velocity distribution function is a local equilibrium state, a class of high resolution MUSCL–type KFVS schemes are presented to approximate the Euler equations of gas dynamics with a general EOS. The schemes are finally applied to several test problems for a general EOS. In comparison with the exact solutions, our schemes give correct location and more accurate resolution of discontinuities. The extension of our idea to multidimensional case is natural.

Mathematics subject classification: 65M06,76M20, 76N15.
Key Words: KFVS method, The Euler equations, General equation of state.

1. Introduction

The development of KFVS schemes and BGK–type schemes for compressible flow simulations has attracted much attention and becomes mature in the past few years. The gas-kinetic schemes have provided robust and accurate numerical solutions for various unsteady compressible flows (see [3, 11, 13, 14, 15, 16, 17, 18, 20]).

Numerical simulation of compressible flows of real gases has been conducted by several authors in [5, 6, 7, 8, 9, 12, 22]. Colella and Glaz in [5] obtained an exact Riemann solver for real gases. Glaister in [7] presented an approximate linearised Riemann solver for the Euler equations with a general EOS. Grossman and Walters [8], and Liou, van Leer and Shuen [9], Vinokur and Montagne [22] extended flux–vector splitting and flux–difference splitting to the Euler equations with general pressure laws.

Most of the previous methods would require a computation of the pressure law and its derivatives, or a Riemann solver. This is costly and problematic when there is no analytic expressions of the pressure law. Recently Coquel and Perthame [6] introduced an energy relaxation theory for the Euler equations of real gas. Their method does not need computations of derivatives of the pressure law or a Riemann solver. In [12], Montarnal and Shu studied the implementation of this relaxation method with high order WENO schemes for real gases.

As we know, the gas distribution function for a local equilibrium state plays an important role in the construction of gas-kinetic schemes, and the Maxwellian distribution function, or

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the Maxwell-Boltzmann distribution can be used to recover the Euler equations with a γ–gas law. However, one cannot use these equilibrium distribution functions to recover the Euler equations with a general EOS. In order to do it, a new equilibrium distribution with two parameters of temperature approximation will be introduced in this paper to recover the Euler equations with general pressure laws. These parameters can be decided uniquely by macroscopic variables. Moreover, using the well-known connection that the Euler equations of motion are the moments of the Boltzmann equation whenever the velocity distribution function is a local equilibrium state, we will also give a class of high resolution KFVS methods to approximate the Euler equations of gas dynamics with a general EOS. They do not depend on the particular expression of the equation of state, besides without derivatives of the pressure law or any Riemann solvers. Finally, several test problems for some special EOSs will be solved by the present schemes to show their performance.

The paper is organized as follows. In next section we will recall the connection between the Boltzmann and the Euler equations, and introduce a new equilibrium distribution function to recover the Euler equations with a general EOS, which can be considered as a generalized local Maxwellian distribution function. In Section 3, a class of high resolution KFVS schemes are presented based on the local equilibrium distribution function introduced in Section 2, and the van Leer's interpolation method. In Section 4, we give several numerical experiments on a standard shock reflection test problem and two shock-tube problems for three different EOSs to show the performance of the current schemes. We conclude the paper with a few remarks in section 5.

2. A New Local Equilibrium Distribution

The Euler equations governing inviscid compressible fluid flows can be described as equations for the mass, momentum and energy densities, $\rho(x,t)$, $\rho(x,t)u(x,t)$, and $E=\frac{1}{2}\rho u^2+\rho e(x,t)$, i.e.

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}=0, \tag{2.1}$$

where

$$U=[\rho,\ \rho u,\ E]^T,\quad F(U)=\left[\rho u,\rho u^2+p,u(E+p)\right]^T. \tag{2.2}$$

In the above ρ, u, p, and e represent the density, velocity, pressure, and the specific internal energy density, respectively.

The system (2.1) is not complete. We should consider an additional equation, i.e. the EOS, which is a macroscopic thermodynamic relationship specific to each particular fluid, and assume here that it can be written in the form

$$p=p(\rho,e). \tag{2.3}$$

Moreover, the function $p(\cdot,\cdot)$ will be assumed to satisfy conditions which ensure that the system (2.1) is hyperbolic. In the case of an ideal gas, (2.3) becomes

$$p=(\gamma-1)\rho e, \tag{2.4}$$

here γ is the ratio of specific heat capacities of the fluid.

Flow motion can also be described from viewpoint of particle motion, or the statistical description of a fluid. Due to the large number of particles in small volume in common situations, to follow each one is impossible. Instead, a continuous distribution function $f(x_i,t,v_i)$ is used to describe the probability of particles to be located in a certain velocity interval, and to approximate usually the particle number density at a certain velocity in hydrodynamics. The velocity distribution function f is a basic unknown in the kinetic theory of gases, and obeys a particle conservation law, known as the Boltzmann equation, which is given by

$$\frac{\partial f}{\partial t}+v\cdot\frac{\partial f}{\partial x}=J(f,f), \tag{2.5}$$