Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 6, pp. 1438-1460

DOI: 10.4208/aamm.OA-2016-0115
December 2017

Analysis of Mathematics and Numerical Pattern Formation in Superdiffusive Fractional Multicomponent System

Kolade M. Owolabi and Abdon Atangana*

Institute for Groundwater Studies, Faculty of Natural and Agricultural Sciences, University of the Free State, Bloemfontein 9300, South Africa

Received 9 September 2016; Accepted (in revised version) 20 January 2017

Abstract. In this work, we examine the mathematical analysis and numerical simulation of pattern formation in a subdiffusive multicomponents fractional-reaction-diffusion system that models the spatial interrelationship between two preys and predator species. The major result is centered on the analysis of the system for linear stability. Analysis of the main model reflects that the dynamical system is locally and globally asymptotically stable. We propose some useful theorems based on the existence and permanence of the species to validate our theoretical findings. Reliable and efficient methods in space and time are formulated to handle any space fractional reaction-diffusion system. We numerically present the complexity of the dynamics that are theoretically discussed. The simulation results in one, two and three dimensions show some amazing scenarios.

AMS subject classifications: 35A05, 35K57, 65L05, 65M06, 93C10

Key words: Asymptotically stable, coexistence, Fourier spectral method, numerical simulations, predator-prey, fractional multi-species system.

1 Introduction

Multicomponents system where species share common resources have drawn much attention of researchers dated to the pioneering work of Holt [33] based on apparent competition [41, 45, 56, 61]. Over the years, reaction-diffusion systems arise from the study of multi-species Lotka-Volterra interactions such as the predator-prey, competition, mutuality and food-chain models have been the subject of great interests [15, 18, 21, 25, 33, 37, 38, 41, 51]. Recently, many authors studied three-species population dynamics with various

*Corresponding author.
Emails: mkowolax@yahoo.com (K. M. Owolabi), abdonatangana@yahoo.fr (A. Atangana)

functional responses, impulsive effects, time delays and stage-structures (see, for example, [16,22,23,28,32,52,56,63]) and obtained some results on permanence, global existence of solution, asymptotic stability, or instability of the nontrivial states and periodicity of solutions. In the present paper, we extension to the study of population dynamics from two species predator-prey model to a three species space fractional reaction-diffusion systems consisting of two preys and one predator with impulsive effect.

Many ecological processes are governed by the fact that they experience a sudden change of state at certain moment of time. These processes evolve as a result of short-time perturbation with a very small time-lag in comparison with the period of the process. In natural sense, it is reasonable to assume that perturbation arise in the form of impulse, and most biological phenomena involving pharmacokinetics systems, thresholds and optimal control models exhibit some kind of impulsive effects. Various work has been done where impulsive control strategy is utilized to investigate the behaviour of the predator-prey dynamics, see [34, 53, 57, 58, 61, 62, 64] and references therein.

The ecological implication of the systems involve three species **U**, **V** and **W**, where **W** is the predator that feeds on both preys **U** and **V**. This type of ecological systems best describe the spatial interactions between the prey and predators among many biological species. Let $u(x,t)$, $v(x,t)$ and $w(x,t)$ be the corresponding scaled density functions of **U**, **V** and **W**, respectively at position x and time t. Then the ecological equations governing their density functions are given in the form of Lotka-Volterra type which consist of two preys and one predator with impulsive effect. A coupled fractional reaction-diffusion system of n ($n \le 3$, n integer) species which interact in a nonlinear fashion and diffuse may be modelled by the equations

$$u_t - D_1\Delta^{\eta/2}u = f(u,v,w), \tag{1.1a}$$
$$v_t - D_2\Delta^{\eta/2}v = g(u,v,w) \quad \text{in } \Omega\times[0,\infty), \quad t \neq T(\epsilon), \tag{1.1b}$$
$$w_t - D_3\Delta^{\eta/2}w = h(u,v,w), \tag{1.1c}$$
$$B[u] = B[v] = 0, \quad B[w] = \mathcal{S} \quad \text{on } \partial\Omega\times[0,\infty), \quad t = T(\epsilon), \tag{1.1d}$$
$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad u(x,0) = u_0(x), \quad (x,t)\in\Omega, \tag{1.1e}$$

where the local kinetics are given as

$$f(u,v,w) = \tau_1 u\left(1-\frac{u}{\kappa_1}\right) - \frac{\alpha_1\varphi_1 uw}{\beta_1+u+\gamma_1 w}, \tag{1.2a}$$
$$g(u,v,w) = \tau_2 v\left(1-\frac{v}{\kappa_2}\right) - \frac{\alpha_2\varphi_2 vw}{\beta_2+v+\gamma_2 w}, \tag{1.2b}$$
$$h(u,v,w) = \left(\frac{\rho_1\alpha_1 u}{\beta_1+u+\gamma_1 w} - \psi_1\right)\varphi_1 w + \left(\frac{\rho_2\alpha_2 v}{\beta_2+v+\gamma_2 w} - \psi_2\right)\varphi_2 w, \tag{1.2c}$$

and Δ denotes the Laplacian operator in one, two or more dimensional space, $\Delta^{\eta/2}$ stands