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A COLLOCATION METHOD FOR THE CONDUCTIVITY PROBLEM WITH DISCONTINUOUS COEFFICIENT*

Yang Hongyong(杨洪勇)   Yang Xin(杨昕)   Wang Yanbo (王彦博)

Abstract *In this paper, a new collocation BEM for the Robin boundary value problem of the conductivity equation* $\nabla(\gamma\nabla u) = 0$ *is discussed, where the* γ *is a piecewise constant function. By the integral representation formula of the solution of the conductivity equation on the boundary and interface, the boundary integral equations are obtained. We discuss the properties of these integral equations and propose a collocation method for solving these boundary integral equations. Both the theoretical analysis and the error analysis are presented and a numerical example is given.*

Key words *piecewise constant coefficient, Robin boundary condition, boundary integral equations, collocation method.*

AMS(2000)subject classifications 45P05, 45Q05, 65N38, 35Q60

1 Introduction

Let $\Omega \subset R^2$ be a bounded simply connected domain with a smooth boundary $\partial\Omega$. Suppose that D_j, $j = 1, 2, \cdots, N$ are the simply connected subdomains of Ω with C^2 smooth boundary, which satisfy

$$\overline{D}_j \cap \overline{D}_k = \emptyset, \qquad j \neq k.$$

We consider the Robin boundary value problem for the conductivity equation with discontinuous coefficient

$$\nabla(\gamma(x)\nabla u(x)) = 0, \quad x \in \Omega, \tag{1.1}$$

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where the conductivity coefficient

$$\gamma(x) = \begin{cases} 1, & x \in \Omega \setminus \bigcup_{j=1}^{N} D_j, \\ k_j, & \in D_j \end{cases}$$

and $k_j \neq 1$, $j = 1, 2, \cdots, N$ be positive constants.

On the boundary $\partial\Omega$, the Robin boundary condition is given as

$$\frac{\partial u}{\partial \nu}(x) + \lambda u(x) = g(x), \qquad x \in \partial\Omega, \tag{1.2}$$

where $g \in H^{\frac{1}{2}}(\partial\Omega)$ and λ is a positive constant, ν is the unit outer normal vector on the boundary $\partial\Omega$.

By the standard techniques in elliptic partial differential equation, it can be easily proved that there exists a unique solution $u \in H^1(\Omega)$ for the problem (1.1),(1.2) if we assume that $\lambda > 0$ ([4],[6]).

The conductivity equation (1.1) is used to describe a conductive body which contains the inclusions of different materials. The conductivities of the body and the inclusions are 1 and k_j, $j = 1, 2, \cdots, N$ respectively. From the theory of partial differential equation, it is known that the regularity of the solution is at most $H^1(\Omega)$ and the derivatives of u must have some jumps along the interface boundary of the two different materials. There are quite a lot of research results on the numerical simulation of the equation by finite element method. We can refer to [1],[2]. The purpose of this paper is to provide a different method–BEM for solving this problem. The main difficulty for applying BEM to this problem is that we do not know the exact formula for the fundamental solution of the conductivity equation (1.1). The key of our method is based on the integral representation formula of the solution of the (1.1) which is proved in [6] for proving the uniqueness of the inverse conductivity problem within the class of disks. We found that this way can be easily realized and we can get quite nice numerical results. Especially, the discontinuous of the derivatives of the solution can be presented very well in the numerical simulation results. This is very important for the parameter determination problems. We will discuss this topic in details in our forthcoming paper. A related work can be found in [7].

It should be remarked that our method works for the conductivity problem in which the interface boundaries are Liptschitz curves.

This paper is organized as follows. In section 2, we will state and prove our main results. The numerical simulation results will be presented in section 3, and some conclusion remarks will be given in section 4.

2 Main results

2.1 The case $N = 1$