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A (2+1)-Dimensional Dispersive Long Wave Hierarchy and its Integrable Couplings

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Abstract

Under the frame of the (2+1)-dimensional zero curvature equation and Tu model, the (2+1)-dimensional dispersive long wave hierarchy is obtained. Furthermore, the loop algebra is expanded into a larger one. Moreover, a class of integrable coupling system for dispersive long wave hierarchy and (2+1)-dimensional multi-component integrable system will be investigated.

Keywords: (2+1)-dimensional zero curvature equation; integrable coupling; loop algebra; multi-component hierarchy.
Mathematics subject classification: 35Q51

1. Introduction

Integrable systems and soliton theory have been receiving more recognition in the mathematical and physics communities. A central and very important topic in the study of integrable system is to search for new Lax or Liouville systems which are associated with certain evolution equations with physical meaning. In [1], Tu proposed a new method based on the analysis of loop algebra. By using the loop algebra, some well-known integrable Hamiltonian hierarchies were worked out, such as AKNS hierarchy, KN hierarchy, WKI hierarchy, BPT hierarchy [1-4]. In order to produce multi-component integrable systems, Guo and Zhang [5-9] constructed a class of multi-component loop algebra $\tilde{G}_M$, and proposed some multi-component integrable systems. Zhou expressed the (2+1)-dimensional three-wave equation as a (2+1)-dimensional zero curvature equation whose almost-periodic solutions are obtained [10]. In this paper, a (2+1)-dimensional dispersive long wave hierarchy is presented by using the (2+1)-dimensional zero curvature equation. Further, we expand the loop algebra into a larger one. We will also investigate a class of integrable coupling system and multi-component integrable systems.

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2. A (2+1)-dimensional integrable system

Denote $\frac{\partial}{\partial z}=\frac{\partial}{\partial y}-\frac{\partial}{\partial x}, \frac{\partial}{\partial w}=\frac{\partial}{\partial t}-\frac{\partial}{\partial x}$. Then zero curvature equation

$$U_w - V_z + [U,V] = 0 \tag{2.1}$$

can be written as a (2+1)-dimensional form

$$U_t - V_y + [U,V] + V_x - U_x = 0, \tag{2.2}$$

which is regarded as the compatibility of the Lax pairs

$$\begin{cases} \varphi_y = \varphi_x + U\varphi, \\ \varphi_t = \varphi_x + V\varphi, \quad \lambda_t = 0. \end{cases} \tag{2.3}$$

We wish to use (2.3) or (2.2) to produce a hierarchy of soliton equations systematically, instead of a single equation. In what follows, we consider the dispersive long wave isospectral problem. Take the loop algebra, see, e.g., [1]

$$\begin{cases} e_1(n)=\lambda^n\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad e_2(n)=\lambda^n\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}, \quad e_3(n)=\lambda^n\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, \\ [e_1(m),e_2(n)]=2e_2(m+n), \quad [e_1(m),e_3(n)]=-2e_3(m+n), \\ [e_2(m),e_3(n)]=e_1(m+n), \\ \deg(e_k(n))=n, \ k=1,2,3. \end{cases} \tag{2.4}$$

Consider an isospectral problem

$$\begin{cases} \varphi_y = \varphi_x + U\varphi, \quad \lambda_t = 0, \\ U = \frac{1}{2}e_1(1) - \frac{1}{2}qe_1(0) - re_2(0) + e_3(0). \end{cases} \tag{2.5}$$

Set

$$V = \sum_{m\geq 0}(a_m e_1(-m) + b_m e_2(-m) + c_m e_3(-m)).$$

From the stationary zero curvature equation

$$V_y - V_x = [U,V], \tag{2.6}$$

one arrives at the recursion relation as follows:

$$\begin{cases} a_{my} - a_{mx} = -b_m - rc_m, \\ b_{my} - b_{mx} = b_{m+1} - qb_m + 2ra_m, \\ c_{my} - c_{mx} = -c_{m+1} + qc_m + 2a_m, \\ a_0 = \alpha = const \neq 0, \quad b_0 = c_0 = 0, \quad a_1 = 0, \quad b_1 = -2\alpha r, \quad c_1 = 2\alpha, \\ a_2 = 2\alpha r, \quad b_2 = -2\alpha[(r_y - r_x) + qr], \quad c_2 = 2\alpha q, \\ b_3 = -2\alpha[(r_y - r_x + qr)_y - (r_y - r_x + qr)_x + q((r_y - r_x + qr) + 2r^2], \\ c_3 = 2\alpha(q_x - q_y + q^2 + 2r), \quad a_3 = 2\alpha(r_y - r_x + 2qr)\cdots \end{cases} \tag{2.7}$$