SPACE-TIME ARIMA MODELING FOR REGIONAL PRECIPITATION FORECASTING*

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Abstract

An aggregate regional forecasting model class belonging to the general family of Space-Time Auto Regressive Moving Average (STARMA) processes is investigated. These models are characterised by autoregressive and moving average terms lagged in both time and space. The paper demonstrates an iterative procedure for building a STARIMA model of a precipitation time series. Eleven raingage sites located in a watershed in southern Ontario, Canada, sampled at 15-day intervals for the period of 1966 to 1980 are used in the numerical analysis. The identified model is STMA(l_3). The model parameters are estimated by the polytope technique, a nonlinear optimization algorithm. The developed model performed well in regional forecasting and in describing the spatio-temporal characteristics of the precipitation time series.

§ 1. Introduction

Time series analysis for modeling and forecasting of hydrologic variables is a valuable and important step in water resources planning and management. In hydrology, the selection of models for analysis of time series is essentially based on simulation and statistical decision theory (Salas et al., 1980). A flexible class of empirical models is the general family of autoregressive moving average (ARMA) processes (Box and Jenkins, 1976). These models have proven very useful in hydrologic analyses (Hipel et al., 1977), but since they are univariate, they are applicable only to single series of data. In constructing an appropriate dynamic stochastic model for a given one-dimensional time series a three-stage iterative procedure is usually followed. This is commonly referred to as the Box-Jenkins method (Box and Jenkins, 1976).

There is an increasing interest in hydrology to develop empirical spatio-temporal models in the context of hydrologic regional analysis and forecasting (Perry and Aroian, 1979; Pfeifer and Deutsch, 1981; Mohamed, 1985). Naturally, an alternative to univariate time series modeling is multivariate time series modeling (Anderson, 1958; Hannan, 1970). An appropriate class of formal models for describing space-time hydrologic data sets is again provided by linear stochastic difference equations. These models attempt to simultaneously describe a set of N

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observable time series. When these N time series represent spatially-located data, the interrelationships and the spatial correlation (Cliff and Ord, 1973) between the different spatial data sets can be taken into account and thus a better system description should result. The spatial domain is incorporated in the modeling procedure by using a hierarchical ordering of the spatial neighbors of each location (Besag, 1974).

The objective of this paper is to develop a spatio-temporal precipitation time series model from the general class of space-time autoregressive moving average (STARMA) processes suitable for regional hydrologic analysis and forecasting. The methodology is essentially an extension of the Box-Jenkins model-building procedure to take into account the spatial effect of existing time series over hydrologically homogeneous areas. In other words, the watershed is considered to be homogeneous with respect to the spatial and temporal variation of physical and hydrologic characteristics, climatic variables and system response.

The paper is organized as follows. In Section 2 the three-stage iterative model-building procedure is developed. Specifically, in the identification stage a preliminary analysis of the data is performed to select a tentative spatio-temporal model. The order of the STARMA model is chosen based on the estimation of space-time autocorrelation (STACF) and space-time partial autocorrelation functions (STPACF). The second stage covers the parameter estimation of the selected tentative STARMA model. The third stage of diagnostic checking deals with the adequacy of the fitted STARMA model, since the goal remains to obtain an adequate but parsimonious model with the smallest number of parameters meeting certain statistical accuracy criteria. If any inadequacy is found, the three-stage iterative procedure is repeated. Section 3 describes the data environment and, finally, in Section 4 a hydrologic application is discussed.

§ 2. Starima Model-Building Procedure

An extension of the Box-Jenkins univariate ARIMA process into the spatial domain leads to the formulation of the general family of STARIMA models (Martin and Oeppen, 1975; Cliff and Orb, 1975; Bennett, 1975; Hooper and Hewings, 1981; Pfeifer and Deutsch, 1980; Mohamed, 1985). The general form of the STARMA model is

$$y_{it}=a_{it}+\sum_{s=0}^{l}\sum_{k=1}^{p}\phi_{sk}L_s y_{i(t-k)}-\sum_{s=0}^{m}\sum_{k=1}^{q}\theta_{sk}L_s a_{i(t-k)}, \tag{1}$$

where y_{it} is the time series at time t and at site i, $i=1, 2, \cdots, N$, p and q are the temporal orders of the AR and MA terms, respectively, l and m are the spatial orders of the AR and MA terms, respectively, ϕ_{sk} and θ_{sk} are parameters of the AR and MA terms, respectively, with spatial lag s and temporal lag k, L_s is the spatial lag operator of lag s, and a_{it} are normally independently distributed white noise residuals with

$$E[a_{it}]=0$$

and

$$E[a_{it}a_{j(t-k)}]=\begin{cases}\sigma^2, & \text{for } i=j,\ k=0,\\ 0, & \text{otherwise.}\end{cases} \tag{2}$$

The spatial lag operator L_s of the space-time model may be defined such that

$$L_s y_{it} = \sum_{j=1}^{N} w_{ijs} y_{jt} \quad \text{for } s>0, \tag{3}$$

where w_{ijs} are a set of weights scaled so that

$$\sum_{j=1}^{N} w_{ijs} = 1 \tag{4}$$

for all i and w_{ijs} nonzero only for i and j sites being s th order neighbors. For $s=0$, equation (3) becomes $L_0 y_{it} = y_{it}$. The weights may reflect physical characteristics of the observed time series and follow a hierarchical ordering of spatial neighbors based on distances between the observation sites in the watershed.

The STARIMA model class expresses y_{it} as a weighted linear combination of past observations and errors, which may be lagged both in space and time. These models can be used for regional forecasting. The STARMA model class collapses into the ARMA model class in the absence of spatial correlation. Two special subclasses of the STARMA model are of note. Models that contain no autoregressive term ($p=0$) are referred to as space-time moving average (STMA) processes of order m in space and q in time

$$y_{it} = a_{it} - \sum_{s=0}^{m} \sum_{k=1}^{q} \theta_{sk} L_s a_{i(t-k)}. \tag{5}$$

When $q=0$, the class is referred to as space-time autoregressive (STAR) process of order l in space and p in time

$$y_{it} = a_{it} + \sum_{s=0}^{l} \sum_{k=1}^{p} \phi_{sk} L_s y_{i(t-k)}. \tag{6}$$

2.1. *Identification of the* STARIMA *model*

Identification of a tentative space-time model is carried out through the analysis of historical data. For a given data set, both the degree of nonstationarity and the order and type of representative models can be decided by studying the shape of the appropriate correlation functions. A brief description of the STACF and STPACF and their characteristics for identifying STAR, STMA and STARIMA models follows.

Space-Time Autocorrelation Function (STACF). In a multivariate framework the STACF expresses the covariance between random variables lagged both in time and space. An estimate for the STACF at spatial lag s and time lag k is given by the following equation:

$$r_{sk} = \frac{\sum_{t=k+1}^{T} \sum_{i=1}^{N} \left[z_{it} \left(\sum_{j=1}^{N} (w_{ijs} z_{j(t-k)}) \right) \right]}{\left[\sum_{t=1}^{T} \sum_{i=1}^{N} z_{it}^2 \right]^{1/2} \left[\sum_{t=1}^{T} \sum_{i=1}^{N} \left(\sum_{j=1}^{N} (w_{ijs} z_{jt}) \right)^2 \right]^{1/2}}, \tag{7}$$

where $z_{it} = y_{it} - \bar{y}$ with $\bar{y}$ being an estimate of the space-time grand mean given by

$$\bar{y} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it}. \tag{8}$$

Other equations have also been used to estimate STACF (Martin and Oeppen, 1975; Pfeifer and Deutsch, 1980), but the results have been unsatisfactory either due to smoothing effects caused by the spatial lag operator L_s or due to a change in one of the means with changes in the spatial lag, or even due to variations in the normalizing factor with the time lag k.

Space-Time Partial Autocorrelation Function (STPACF). The STPACF may be expressed by extending the "conventional" definition of Kendall and Stuart (1967) to the space-time series. To compute the partial autocorrelations it is necessary to calculate the symmetric space-time autocorrelation matrix C_v (Fig. 2). This matrix estimates the intercorrelations between all pairs of lagged variables $w_{ijh}z_{j(t-g)}$ and $w_{ijs}z_{j(t-k)}$, where $h, s=0, 1, \cdots, l$ are the spatial lags, $g, k=1, 2, \cdots, p$ are the time lags, and $i, j=1, 2, \cdots, N$ refer to the spatially located variables. These correlations may be expressed by the extended notation $[r_{hgsk}]$ and the matrix C_v may be computed by repeatedly solving the following equation:

$$r_{hgsk}=\frac{\sum_{t=v+1}^{T}\sum_{i=1}^{N}\left[\left[\sum_{j=1}^{N}(w_{ijh}z_{j(t-g)})\right]\left[\sum_{j=1}^{N}(w_{ijs}z_{j(t-k)})\right]\right]}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\sum_{j=1}^{N}(w_{ijh}z_{jt})\right]^2\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\sum_{j=1}^{N}(w_{ijs}z_{jt})\right]^2\right]^{1/2}}, \tag{9}$$

where $v=\max(g, k)$. Thus the space-time partial correlation between z_{it} and $w_{ijs}z_{j(t-k)}$, say ψ_{oosk}, is given by

$$\psi_{oosk}=\frac{-c_{oosk}}{(c_{oooo}c_{sksk})^{1/2}}, \tag{10}$$

where c_{oosk} is the cofactor of the estimated STACF $[r_{oosk}]$ in the correlation determinant $|C|$. An advantage of this method is that the order in which the autocorrelations are written is insignificant, since each partial autocorrelation is computed by assigning certain fixed values to all other lagged terms. If a time series is nonstationary it would require temporal and/or spatial differencing to achieve stationarity. For example, the first order temporal and spatial difference operators are given by

$$\nabla_T y_{it}=y_{it}-y_{i(t-1)} \tag{11}$$

and

$$\nabla_s y_{it}=y_{it}-\sum_{j=1}^{N} w_{ijs}y_{jt}.$$

STARMA processes are characterized by distinct STACF and STPACFs. Specifically, a STAR (l, p) process exhibits autocorrelations that decay in space and time, and partial autocorrelations that cut off after p lags in time and l lags in space. Alternatively, a STMA(m, q) process exhibits autocorrelations that cut off after m^- lags in space and q lags in time and partials that decay exponentially both in space and time. Finally, the STARMA (l, p, m, q) models are characterized by STACF and STPACFs that both tail off in time and space. Table 1 summarizes the characteristics of the theoretical STACF and STPACFs.

2.2. *Estimation of the* STARIMA *model*

Estimates of the parameters ϕ_{sk} and θ_{sk} of the tentative model from the STARMA family of time series models can be computed by minimizing the residual sum of squares:

$$S(\phi, \theta)=a^T a=\sum_{i=1}^{N}\sum_{t=1}^{T}a_{it}^2. \tag{12}$$

The adopted Box-Jenkins modeling procedure requires that the errors a_{it} should be pure white noise. Since the random errors a_{it} are unobservable, a recursive scheme is necessary to calculate the a_{it} from the observed series z_{it}. The commonly used equations result from equation (1) and are given by

$$a_{it} = z_{it} - \sum_{s=0}^{l} \sum_{k=1}^{p} \phi_{sk} L_s z_{i(t-k)} + \sum_{s=0}^{m} \sum_{k=1}^{q} \theta_{sk} L_s a_{i(t-k)}. \tag{13}$$

In the Box–Jenkins time series modeling approach the parameters ϕ_{sk} and θ_{sk} are usually estimated by the maximum likelihood (ML) method (Box and Jenkins, 1976; Hipel et al., 1977; Pfeifer and Deutsch, 1980). However, the accuracy of the conditional ML estimation directly depends on the record length T. Moreover, only the STAR models are linear, whereas the STARMA and the STMA models are nonlinear in form. Any nonlinear optimization technique such as gradient or linearization methods, could then be used to estimate the parameters of the models. In this paper the polytope method (Nelder and Mead, 1965) is employed from a package of nonlinear optimization techniques (Birta, 1983).

The polytope method, which is frequently referred to as the simplex method of Nelder and Mead (1965), is an example of a heuristic procedure for solving the unconstrained function minimization problem. The process begins with the specification of a regular simplex which is defined in terms of $(n+1)$ points in n–space (hence in 2–space the simplex is simply a triangle). Through a sequence of operations referred to as reflection, expansion and contraction, the simplex changes shape and moves through the parameter space until it (hopefully) encompasses, and then contracts upon, the minimizing argument x^*. Each basic step begins with a particular simplex characterized by its vertices $x_1, x_2, \cdots, x_n, x_{n+1}$ and ends with a new simplex whose shape and location have been altered in response to the local topology of the function (Note: throughout this discussion, if v is a vector variable, then the notation v_j is used to denote a particular occurrence of the vector v rather than its jth component). The process can be terminated either when the vertices of the simplex become sufficiently clustered or when the function values at the vertices are all within a prescribed tolerence.

2.3. *Diagnostic Checking of the* STARIMA *model*

Diagnostic checking is performed in order to examine any inadequacy in the selected space–time model (Box and Jenkins, 1976). In this paper the residuals STACF and the Port Manteau tests are used to investigate the whiteness of the residuals. The cumulative periodogram test is also used to examine the presence of any periodicities in the residuals. A brief description of the tests follows.

Anderson's Test. This test is used to approximately assess the statistical significance of departures of the residuals autocorrelation from zero. If the residuals are white noise the autocorrelations should have a zero mean and variance–covariance matrix equal to ${}^2I_N$ and all autocovariances at nonzero lags equal to zero. If any of the residuals' autocorrelations are significantly different from zero, then the model building procedure is considered inadequate and should be repeated. A model can then be identified to represent the residuals, which could be incorporated into the original model to obtain a better updated model for the space–time series.

Port Manteau Test. The whiteness of the estimated residuals of the fitted model is tested using the equation:

$$Q = N \sum_{k=1}^{k} r_k^2(\hat{a}), \tag{14}$$

where $r_k^2(\hat{a})$ is the autocorrelation of the residuals a_{it}, N is the sample size and k is the maximum time lag. The quantity Q is approximately chi-square distributed with $(k-p-q)$ degrees of freedom, where p and q refer to the number of parameters in the AR and MA terms, respectively. The adequacy of the identified STARMA model may be checked by comparing the statistic Q with the tabulated chi-square value $x^2(k-p-q)$ for a given level of significance. If $Q < x^2(k-p-q)$, a_{it} is an independent series resulting in an adequate model; otherwise the model is inadequate.

The Cumulative Periodogram Test. This test provides an effective means for the detection of periodic nonrandomness. The cumulative periodogram of a white noise series a_t, $t=1, 2, \cdots, T$, is given by:

$$P(F_k) = \frac{1}{T\sigma_a^2} \sum_{j=1}^{k} I(F_j), \tag{15}$$

where

$$I(F_j) = \frac{2}{T}\left[\left(\sum_{t=1}^{T} a_t \cos 2\pi F_j t\right)^2 + \left(\sum_{t=1}^{T} a_t \sin 2\pi F_j t\right)^2\right] \tag{16}$$

with $F_j = j/N$ and σ^2 being the variance of the residuals. If the residuals contain any periodicities, the cumulative periodogram would show significant deviation from the lines of the confidence limits at the specified level of significance; otherwise the residuals should lie within the confidence limits and should be considered white noise.

§ 3. The Data Base

The data used in this study consist of precipitation time series from eleven ($N=11$) raingage stations spatially located within the Grand River basin in southern Ontario, Canada (Fig. 1). The selection of the data sets is based on the following criteria: adequate spatial distribution to meet the areal coverage needs of the model development procedure; sufficient and complete record length to satisfy the accuracy requirements of the three-stage iterative modeling approach.

Data were available for the period of July 1966 to June 1980. A time step of 15-days is used in this study, which is an accepted time step to preserve the characteristics of the storm events in the precipitation time series. A portion of the data, i.e., 192 values, is used in the model-building procedure, and the remaining 144 values are available for the evaluation and forecasting phase. It should be mentioned that there were a few missing values in the selected time series. In those cases, the normal-ratio method (Linsley, Kohler and Paulhus, 1982), a simple and widely used technique in hydrology, was adopted to "fill up the gaps" in the precipitation data sets.

The watershed, 3480 km^2 in size, is divided into eleven subareas, one for each raingage site, using the Thiessen polygon technique (Fig. 1). No attempt is made to average each time series over the subareas. As a result, every precipitation series is assumed to represent the corresponding subarea and used in the development of the space–time precipitation model. Based on the selected spatial ordering as defined in Section 2, a hierarchical weighting scheme is designed for the eleven raingage stations (Table 2). In this study equal weights are assigned to the spatial precipitation system. This is illustrated in Fig. 3, where the $\|x\|$ weighting matrix W_1 of spatial order one is shown. It should be noticed that each row of Fig. 3 sums to one and that nonzero entries correspond to the designed pattern of Table 2.

§ 4. Regional Precipitation Forecasting

Initial identification of the precipitation time series suggests that the space–time precipitation system is nonstationary. This is indicated by the STACF and STPACFs of the original precipitation series up to spatial lag 3 and time lag 6 shown in Tables 3 and 4, respectively. First differencing in time is then applied to the data sets to achieve stationarity. The sample STACF and STPACFs of the differenced series are presented in Tables 5 and 6, respectively. The autocorrelations are effectively cut off after $k=1$, for $s=0$, 1, 2 and 3, and the partials decay across $k=1, 2, \cdots, 6$ for $s=0$. In other words, the general pattern is one of decay for the STPACF, whereas the STACF cuts off in time after lag one. These patterns suggest that the differenced precipitation series are tentatively identified as an STMA (l_s) model of order one in time and three in space, which takes the form:

$$\nabla_T z_{it} = a_{it} - \sum_{s=0}^{3} \theta_{s1} W_s a_{i(t-1)}, \tag{17}$$

where θ_{01}, θ_{11}, θ_{21} and θ_{31} are the parameters to be estimated. The same model could be referred to as a STARIMA $(0, 1, l_3)$ model by including the first differencing into the model notation.

The parameters of the STMA (l_s) model are estimated by using the previously described polytope estimation algorithm (Nelder and Mead, 1965). The results of the parameter estimation are summarized in Table 7. In particular, three computer runs of the polytope algorithm are made to estimate the four parameters using different initial guesses. The results are the same for all the attempted runs and suggest a global optimum. Table 7 also presents the initial and final estimates of the residual sum of squares S. The three runs show similar performance, since the final and minimized value of S is the same.

In the diagnostic checking stage, the residuals are tested for whiteness and presence of any periodicities. The residuals are generated by incorporating the parameter estimates of the developed STMA(l_3) into the appropriate form of equation (13) for the model-building period. The whiteness of the generated residuals is checked by using the Anderson and Port Manteau tests. For the first test the residual STACFs are estimated and the results, along with the upper bound $1/\sqrt{N}$ for the residuals' standard errors, are presented in Table 8. The results indicate a lack of structure among the STACFs, which suggests that the generated residuals are uncorrelated and consequently white noise.

Similarly, the Port Manteau test is employed to check for the whiteness of the residuals and the results are presented in Table 9. In particular, a comparison of the Port Manteau statistic with the chi-square statistic at 0.05 level of significance indicates that the residuals pass the test, which suggests that the generated residuals can be considered uncorrelated. Finally, the cumulative periodograms of the generated residuals (equations 15 and 16) are used to test for periodicities and are plotted in Fig. 5 along with the confidence limits at the 0.05 level of significance. Since the periodograms fall within these limits, the generated residuals do not contain any periodicities. Based on the above tests the generated residuals are considered uncorrelated and white noise. Therefore, the STMA(l_3) model passes the diagnostic checking stage of the model-building procedure.

It is felt that the parameter $\theta_{31}=0.0007$ is very small and could be omitted. This suggests that an STMA(l_2) model would be used to describe the precipitation time series. Schematically the model is presented in Fig. 4. Estimation of the STMA(l_2) model parameters using the polytope method results in the following estimates: $\theta_{01}=0.9460$, $\theta_{11}=-0.03896$ and $\theta_{21}=0.04127$ with the residual sum of squares being $S=0.1306\times10^7$. The STMA(l_2) model would then have the following form:

$$\nabla_T z_{it}=a_{it}-0.946a_{i(t-1)}+0.03896W_1a_{i(t-1)}+0.04127W_2a_{i(t-1)}. \tag{18}$$

The diagnostic checks based on the study of the STACF, Port Manteau test and cumulative periodogram concluded that the residuals from the STMA(l_2) are white noise and free of periodicities. Therefore, the STMA(l_2) passes the diagnostic checking stage of model-building procedure.

Since the objective of the Box-Jenkins modeling procedure is always to develop a parsimonious model with the smallest number of parameters meeting certain accuracy criteria, the STMA(l_2) model has been accepted as the space-time model to describe the precipitation time series.

Finally, a validation procedure is followed, which is common in time series analysis (Box and Jenkins, 1976). Specifically, the forecasting performance of the developed model is examined for the evaluation period of the remaining 144 values. The mean and variance of the generated residuals for the model-building period are estimated. These statistical parameters are used to randomly generate normally distributed residuals for the evaluation or forecasting period. These residuals have been tested for whiteness and presence of periodicities using cumulative periodograms and the Port Manteau test and have shown to be uncorrelated and white noise. Figs. 6 and 7 show plots of the observed and the computed series for the model development as well as the evaluation period for two raingage data sets. The plots indicate satisfactory performance for the developed space-time precipitation model, which has already passed the diagnostic checks.

§ 5. Summary and Conclusions

This paper discusses the general class of STARMA models and the three-stage iterative procedure to build a space-time precipitation model for regional forecasting. The developed STMA(l_2) precipitation model of equation (18) performs well in describing the spatio-temporal structure of the selected data sets. The computed precipitation series are found to compare well with the corresponding observed series for the model development and the evaluation period (Figs. 6 and 7). This space-time process is considered appropriate in modeling and forecasting hydrologic time series that exhibit spatial correlation and can be used in simulation and regional analyses.

In this Box-Jenkins procedure a crucial component is the identification stage. The sample STACF and STPACFs appear to be quite successful in suggesting a space-time precipitation model, which approximates the information in the data sets with acceptable accuracy. In particular, the type and order of the model, as well as the degree of nonstationarity, were chosen by studying the shape of the autocorrelation functions. Apparently the choice of spatial ordering, which delineates the influence of one zone on another, can affect the form of STACF and STPACFs and, consequently, can lower the adequacy of the identified space-time model. The selected equal weighting scheme of spatial lags reflects the physical properties of the observed system sufficiently well, although eleven stations are not considered a dense raingage network for the size of the selected watershed.

One of the primary objectives of this study remains the development of a parsimonious space-time model. The methodology described in this paper was successful in showing those characteristics, since the developed STMA (l_2) precipitation model is a parsimonious model with the smallest number of parameters that minimize the residual sum of squares.

Table 1 Characteristics of the theoretical STACF and STPACF for STAR, STMA and STARMA models

Model form	STACF	STPACF
STAR (l, p)	Tails off	Cuts off after p time lags, l spatial lags
STMA (m, q)	Cuts off after q time lags, m spatial lags	Tails off
STARMA (l, p, m, q)	Tails off	Tails off

STACF=Space-Time Autocorrelation Function
STPACF=Space-Time Partial Autocorrelation Function
STAR=Space-Time Autoregressive
STMA=Space-Time Moving Average
STARMA=Space-Time Autoregressive Moving Average

Table 2 The hierarchical spatial ordering scheme for the eleven raingage stations

Spatial order	1	2	3
Raingage 1	3	2	4, 6
2	3, 4	1	5, 8, 9
3	1, 2, 4, 5	6, 8	7, 9
4	2, 3, 8	5, 9	1, 6, 7, 10
5	3, 6, 7, 8	4	2, 9, 10, 11
6	5	3, 7	1, 4, 8
7	5, 8, 11	6, 10	3, 4
8	4, 5, 7, 10	3, 9, 11	2, 6
9	10	4, 8	2, 3, 5, 11
10	8, 9, 11	7	4, 5
11	7, 10	8	5, 9

Table 3 Sample STACF of the original precipitation series

Space lag (s) / Time lag (k)	0	1	2	3
1	0.0866	0.0728	0.0756	0.0752
2	0.0338	0.0134	−0.0029	0.0029
3	0.0298	−0.0010	−0.0112	0.0097
4	−0.0269	−0.0378	−0.0442	−0.0468
5	−0.0412	−0.0514	−0.0401	−0.0471
6	−0.0113	−0.0320	−0.0352	−0.0249

Table 4 Sample STPACF of the original precipitation series

Space lag (s) / Time lag (k)	0	1	2	3
1	0.0381	0.0008	−0.0098	0.0060
2	−0.0478	0.0158	−0.0063	0.0307
3	−0.0280	−0.0174	0.0244	0.0178
4	−0.0308	0.0097	0.0156	0.0302
5	0.0159	0.0087	0.0173	−0.0226
6	−0.0272	0.0292	0.0025	−0.0025

Table 5 Sample STACF of the differenced precipitation series

Space lag (s) / Time lag (k)	0	1	2	3
1	−0.4711	−0.3660	−0.3971	−0.3490
2	−0.0207	−0.0180	−0.0320	−0.0358
3	0.0251	0.0081	0.0097	0.0302
4	−0.0220	−0.0112	−0.0192	−0.0298
5	−0.0241	−0.0181	−0.0005	−0.0121
6	0.0488	0.0327	0.0204	0.0384

Table 6 Sample STPACF of the differenced precipitation series

Space lag (s) / Time lag (k)	0	1	2	3
1	0.3736	−0.0092	−0.0212	−0.0067
2	0.2316	0.0001	−0.0299	0.0104
3	0.1646	−0.0155	−0.0139	0.0125
4	0.1111	−0.0109	−0.0081	0.0259
5	0.1008	−0.0133	0.0011	−0.0009
6	0.0549	−0.0009	−0.0004	−0.0148

Table 7 Parameter Estimates of the STMA (l_3) precipitation model

Run	Parameter	Guess	Estimate	Initial S	Final S
1	θ_{01}	0.000	0.9455		
	θ_{11}	0.000	−0.0385		
	θ_{21}	0.000	−0.0417		
	θ_{31}	0.000	0.0007	0.219×10^7	0.130×10^7
2	θ_{01}	0.400	0.9455		
	θ_{11}	0.100	−0.0385		
	θ_{21}	0.200	−0.0417		
	θ_{31}	0.100	0.0007	0.148×10^7	0.130×10^7
3	θ_{01}	0.200	0.9455		
	θ_{11}	−0.100	−0.0385		
	θ_{21}	−0.100	−0.0417		
	θ_{31}	−0.100	0.0007	0.198×10^7	0.130×10^7

S is the residual sum of squares (equation (12))

Table 8 Sample STACF of the generated residuals

Space lag (s) / Time lag (k)	0	1	2	3
1	0.0572	0.0575	0.0620	0.0609
2	0.0128	0.0060	−0.0079	−0.0020
3	0.0084	−0.0093	−0.0171	0.0022
4	−0.0427	−0.0404	−0.0456	−0.0487
5	−0.0514	−0.0501	−0.0385	−0.0447
6	−0.0182	−0.0267	−0.0292	−0.0190

Table 9 Port Manteau test on the generated residuals

Spatial lag (s)	Port Manteau statistics	Chi-square statistic 0.05	Remarks
0	7.52	26.3	Accepted
1	8.21	26.3	Accepted
2	7.50	26.3	Accepted
3	7.5	26.3	Accepted

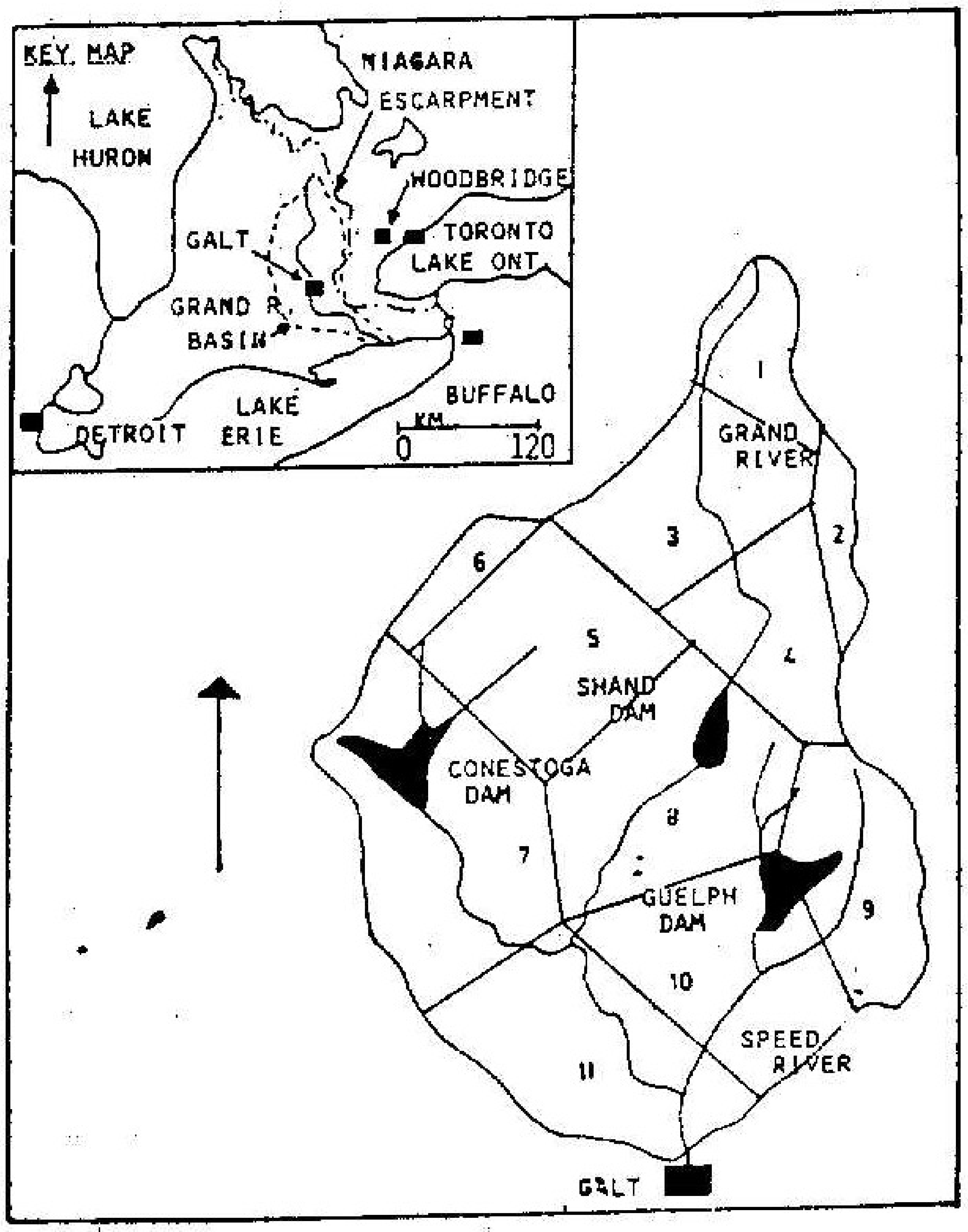

Fig. 1 The watershed and key map

$$
C_y=\begin{array}{cc|cccccccccc}
 & & k=0 & k=1 & & & & & k=p & & & \\
 & & s=0 & s=0 & s=1 & \cdots & s=l & & s=0 & s=1 & \cdots & s=l \\
\hline
g=0 & h=0 & r_{0000} & r_{0001} & r_{0011} & \cdots & r_{00l1} & \cdots & r_{000p} & r_{001p} & \cdots & r_{00lp} \\
 & h=1 & r_{0100} & r_{0101} & r_{0111} & \cdots & r_{01l1} & \cdots & \cdots & \cdots & \cdots & r_{00lp} \\
 & h=2 & r_{1100} & r_{1101} & r_{1111} & \cdots & r_{11l1} & \cdots & \cdots & \cdots & \cdots & r_{11lp} \\
 & \vdots & & & & & & & & & & \\
 & h=l & r_{l100} & r_{l101} & r_{l111} & \cdots & r_{l1l1} & \cdots & \cdots & \cdots & \cdots & r_{l1lp} \\
 & \vdots & & & & & & & & & & \\
 & h=0 & r_{0p00} & \vdots & \vdots & & \vdots & & & & & \vdots \\
 & h=1 & r_{1p00} & \vdots & \vdots & & \vdots & & & & & \vdots \\
g=p & & \vdots & \vdots & \vdots & & \vdots & & & & & \vdots \\
 & h=l & r_{lp00} & r_{lp01} & r_{lp11} & \cdots & r_{lpl1} & \cdots & r_{lp0p} & r_{lp1p} & \cdots & r_{lplp}
\end{array}
$$

Fig. 2 The symmetric space-time autocorrelation matrix C_y

$W_1 =$

	1	2	3	4	5	6	7	8	9	10	11
1			1.0								
2			0.5	0.5							
3	0.25	0.25		0.25	0.25						
4		0.33	0.33					0.33			
5			0.25			0.25	0.25	0.25			
6					1.0						
7					0.33			0.33			0.33
8				0.25	0.25		0.25			0.25	
9										1.0	
10								0.33	0.33		0.33
11							0.5			0.5	

Fig. 3 The weighting matrix W_1 of spatial order one for the 11-raingage system

$$
\begin{bmatrix} s_{11} \\ s_{21} \\ \vdots \\ s_{N1} \\ \hline s_{12} \\ s_{22} \\ \vdots \\ s_{N2} \\ \hline \vdots \\ \hline s_{1T} \\ s_{2T} \\ \vdots \\ s_{NT} \end{bmatrix}
=
\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{N1} \\ \hline a_{12} \\ a_{22} \\ \vdots \\ a_{N2} \\ \hline \vdots \\ \hline a_{1T} \\ a_{2T} \\ \vdots \\ a_{NT} \end{bmatrix}
-
\left[\begin{array}{c|c|c}
0 & 0 & 0 \\ \hline
\begin{matrix} a_{11} \\ a_{21} \\ \vdots \\ a_{N1} \end{matrix} &
\begin{bmatrix} w_{11} & \cdots & w^1_{1N} \\ w^1_{21} & \cdots & w^1_{2N} \\ \vdots & & \vdots \\ w^1_{N1} & & w^1_{NN} \end{bmatrix} \begin{matrix} a_{11} \\ a_{21} \\ \vdots \\ a_{N1} \end{matrix} &
\begin{bmatrix} w^2_{11} & \cdots & w^2_{1N} \\ w^2_{21} & \cdots & w^2_{2N} \\ \vdots & & \vdots \\ w^2_{N1} & \cdots & w^2_{NN} \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{N1} \end{bmatrix} \\ \hline
\vdots & \vdots & \vdots \\ \hline
\begin{matrix} a_{1(T-1)} \\ a_{2(T-1)} \\ \vdots \\ a_{N(T-1)} \end{matrix} &
\begin{bmatrix} w^1_{11} & \cdots & w^1_{1N} \\ w^1_{21} & & w^1_{2N} \\ \vdots & & \vdots \\ w^1_{N1} & \cdots & w^1_{NN} \end{bmatrix} \begin{bmatrix} a_{1(T-1)} \\ a_{2(T-1)} \\ \cdots \\ a_{N(T-1)} \end{bmatrix} &
\begin{bmatrix} w^2_{11} & \cdots & w^2_{1N} \\ w^2_{21} & & w^2_{2N} \\ \vdots & & \vdots \\ w^2_{N1} & \cdots & w^2_{NN} \end{bmatrix} \begin{bmatrix} a_{1(T-1)} \\ a_{2(T-1)} \\ \vdots \\ a_{N(T-1)} \end{bmatrix}
\end{array}\right]
\begin{bmatrix} \theta_{01} \\ \theta_{11} \\ \theta_{21} \end{bmatrix}
$$

Fig. 4 The general form of the STMA (l_2) model

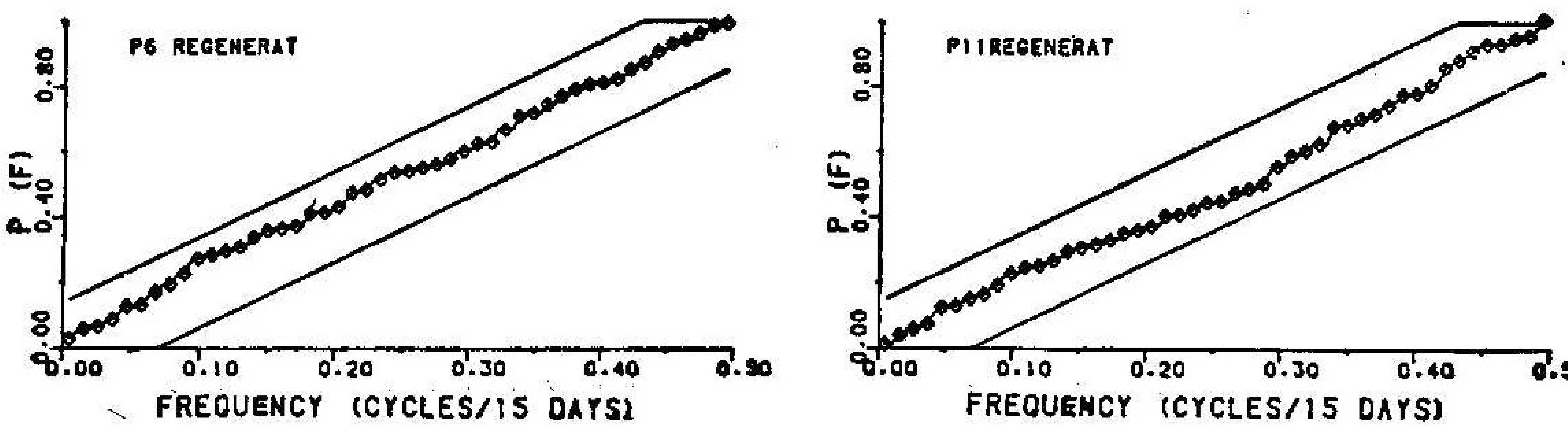

Fig. 5 Cumulative periodograms of generated residuals for two precipitation series

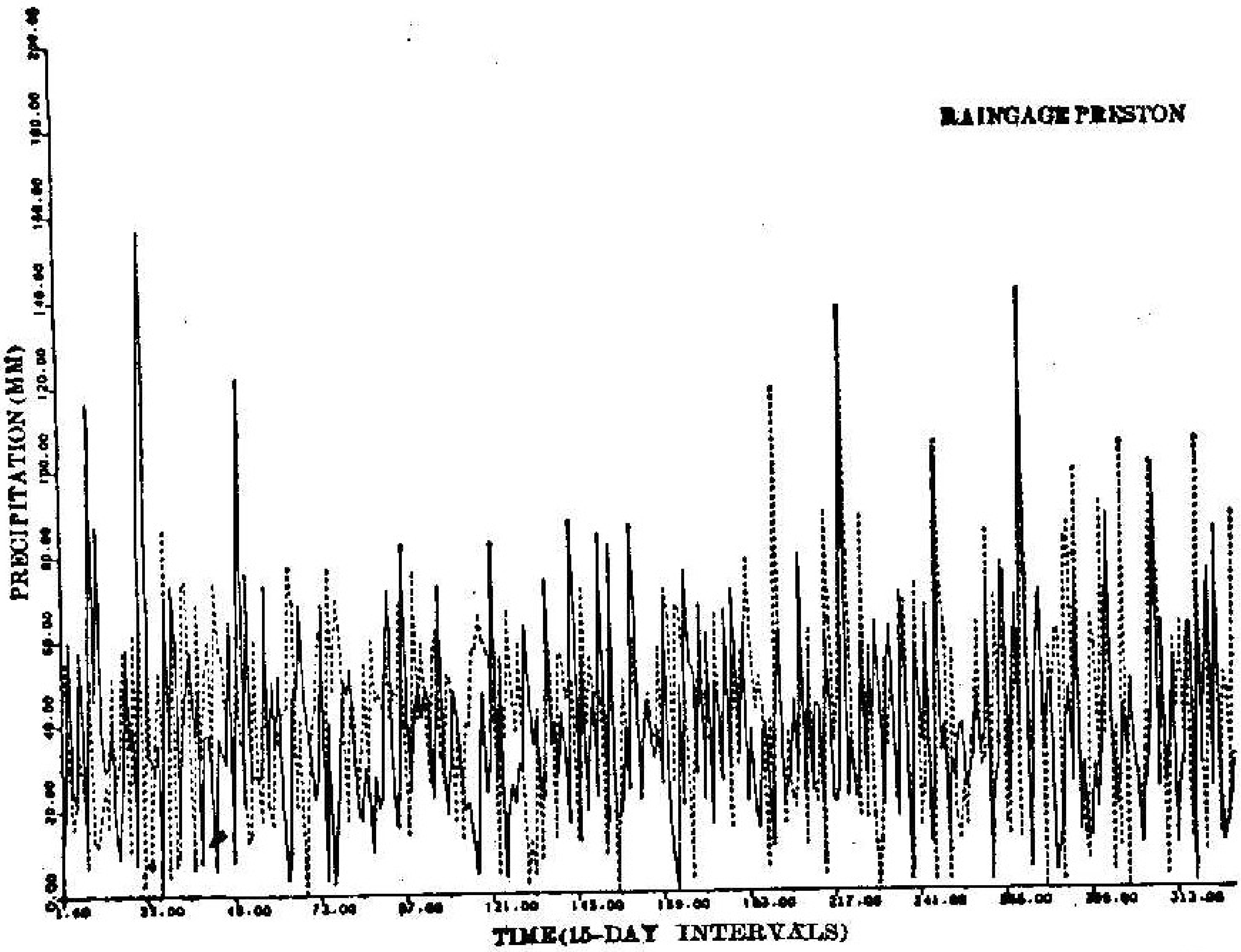

Fig. 6 Observed and generated precipitation series for the Preston Station

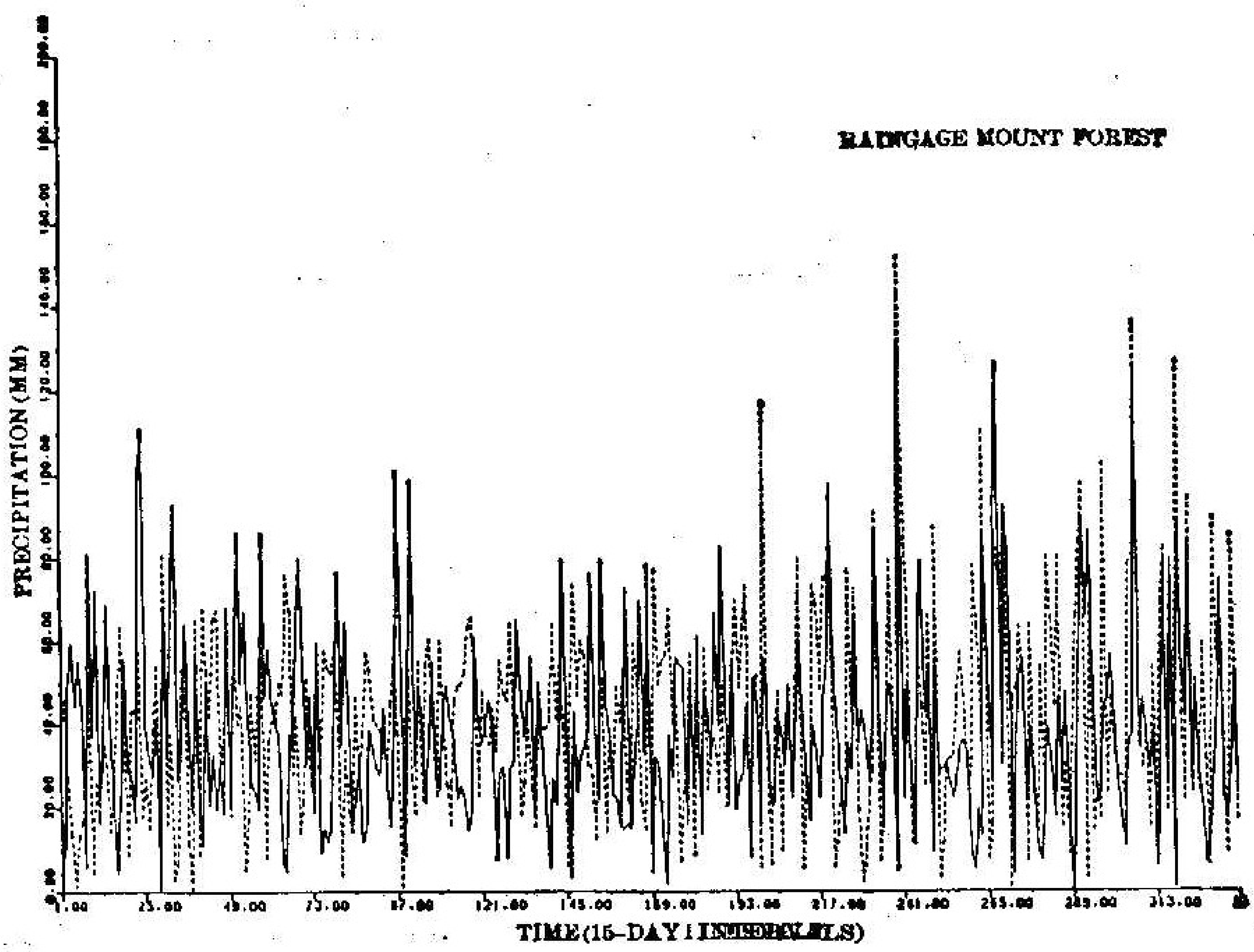

Fig. 7 Observed and generated precipitation series for the Mount Forest Station

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