

Cellular Automaton Model for Fixed Autoblock System

Hua-Liang Zhou¹, Zi-You Gao^{1,*} and Ke-Ping Li¹

¹ *School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, P. R. China.*

Received 4 June 2005; Accepted (in revised version) 27 September 2005

Communicated by Dietrich Stauffer

Abstract. In this work, we propose a kind of cellular automaton model to simulate the railway traffic which is based on NaSch traffic model. The signaling system adopted in this work is the three-aspect fixed autoblock system. We investigate the space-time diagram of traffic flow using our model. The velocities and headways of trains are discussed. The impact of the time interval of trains on train running is also analyzed. Numerical simulation results demonstrate that our cellular automaton model can successfully reproduce some actual railway traffic phenomena.

Key words: Cellular automaton; traffic flow; fixed autoblock system.

1 Introduction

The fixed autoblock system (FAS) has been playing an important role in ensuring train operation safety and improving transport efficiency. In the beginning, two-aspect and three-aspect fixed autoblock systems were very common in railways. Subsequently, European experts proposed the concept of four-aspect, five-aspect and more-aspect FAS to enhance the carrying capacity. With the progress of FAS, the theoretical analysis on FAS has been widely carried out. In order to investigate the performance of FAS, many researchers put forward different models to simulate the train running in FAS. Some professional simulation systems have also been developed [2, 4, 5].

As a simulation tool, a cellular automaton (CA) model has been widely applied to study the traffic flow of highway [1, 3, 8–11]. The advantage of CA model is that it is much simpler and more convenient for computer simulations (it is able to perform several

*Correspondence to: Zi-You Gao, School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, P. R. China. Email: zygao@center.njtu.edu.cn

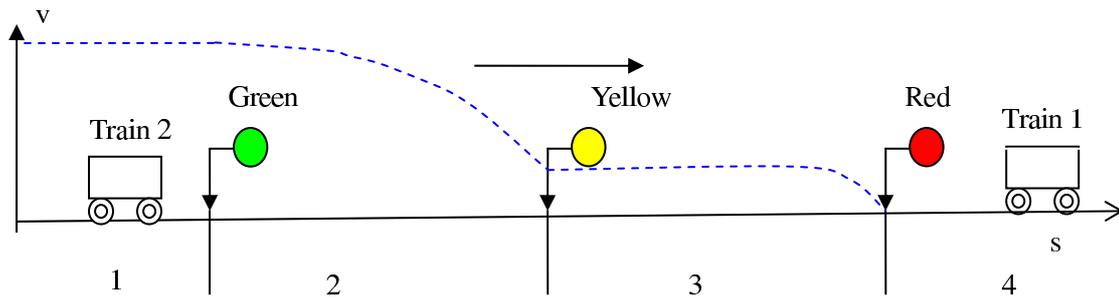


Figure 1: Three-aspect fixed autoblock system.

millions updates in a second [7]). Consequently, we can easily simulate the nonlinear characters in traffic problems using some typical CA models. For example, Wolfram's 184 rules [11], NaSch model [9] and FI model [3]. Recently, K.P. Li et al. proposed a CA model for railway traffic [6]. The signaling system adopted in this model is the moving block system. To our knowledge, this is the first time a CA model has been used to simulate railway traffic using a CA model.

In this paper, we propose another CA railway traffic model based on the NaSch model to simulate the railway traffic. Compared with the CA model proposed by K.P. Li et al., our model adopts the signaling system of FAS. The paper is organized as follows: In Section 2, we introduce the model. In Section 3, the numerical and analytical results are presented. Finally, a conclusion of this approach is presented.

2 The proposed model

2.1 Three-aspect fixed autoblock system

In fixed autoblock systems, the line is divided into blocks by wayside signaling. The length of a block depends on the maximum train speed, the worst-case braking rate and the number of signal aspects which includes a series of additional color displays (or cab indications) used in order to give a driver forewarning of a red signal. Only one train may occupy a block at any time and the presence of a train within a block is usually detected with the use of track circuits. In this study, we investigate a typical three-aspect FAS.

As shown in Fig. 1, the signaling in a three-aspect FAS has three types of colors: red, yellow and green. If the color of signaling in front of a train is green, the train keeps its speed; if yellow, the train slows down; if red, the train must stop before the red signal. The color of signaling changes is based on the trains' positions on the line. For the sake of convenience, we assign to the block k a number, $B(k) = \{1, 0\}$, where $B(k)$ is a block state function. If the block k is occupied by a train, the value of $B(k)$ is 1; if the block is empty, $B(k)$ equals to 0. $Flag(k)$ denotes the signaling color of block k . The updated rules of signaling color are shown in Table 2.1.

Table 1: The updated rules of signaling colour.

Block k does not contain the station	
$B(k) = 1$	$Flag(k) = \text{'red'}$
$B(k) = 0 \ \& \ B(k + 1) = 1$	$Flag(k) = \text{'yellow'}$
$B(k) = 0 \ \& \ B(k + 1) = 0$	$Flag(k) = \text{'green'}$
Block k contains the station	
$B(k) = 1$	$Flag(k) = \text{'red'}$
$B(k) = 0$	$Flag(k) = \text{'green'}$

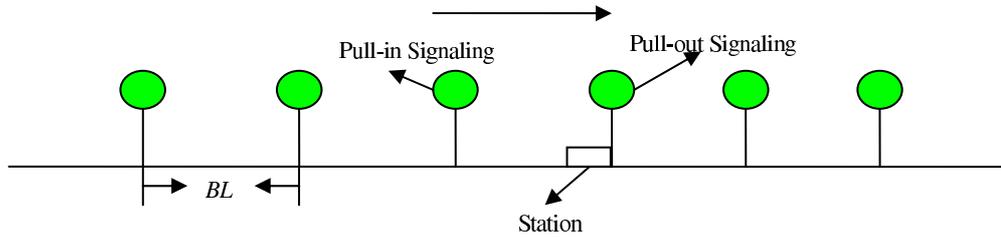


Figure 2: The layout of signaling.

2.2 Cellular automaton model

In this work, we use the CA model to simulate the railway traffic in a three-aspect fixed autoblock system. Our investigation is based on the NaSch traffic model. The lane consists of a single-lane which is divided into L cells of equal size numbered by $i = 1, 2, \dots, L$, and the time is discrete. Each site can be either empty or occupied by a train with integer velocity $v_n = 0, 1, \dots, v_{max}$. The block length is denoted as BL which is larger than the train's braking distance. The layout of signaling is shown in Fig. 2.

Before we introduce the update rules of the system, we define two speed restrictions functions: yellow signal speed restrictions function $v_y(s)$ and red signal speed restrictions function $v_r(s)$. Here s denotes the distance between the train's head and the signaling position in front of the train. Let b be the train's deceleration. If the train faces a yellow signal, the train's speed should be less than or the same as $v_y(s)$. $v_y(s)$ should satisfy Eqs. (2.1)-(2.2).

$$v_y(s)^2 - v_l^2 = 2bs \tag{2.1}$$

$$v_y(s) \leq v_{max}, \tag{2.2}$$

where v_l is the speed restriction when the train reaches the yellow signaling. From Eqs. (2.1)-(2.2), we can get Eq. (2.3),

$$v_y(s) = \min(\sqrt{2bs + v_l^2}, v_{max}). \tag{2.3}$$

If the train passes the yellow signal and faces the red signal, the train's speed should be less than v_l and should become zero before the red signaling. Similarly, we can derive the red signal speed restrictions function $v_r(s)$:

$$v_r(s) = \min(\sqrt{2bs}, v_l). \quad (2.4)$$

The two speed restrictions functions ensure that the train's speed can slow down gradually to v_l before reaching a yellow signal and to 0 before reaching a red signal.

The update rules for implementing the railway traffic are as follows:

STEP 1: update the trains' positions

CASE I: the train n is behind a signaling.

(1) Update speed:

```

If  $Flag == 'green'$ 
     $v_n = \min(v_n + a, v_{max})$ 
Elseif  $Flag == 'yellow'$ 
     $v_n = \min(v_n + a, v_y(s))$ 
Elseif  $Flag == 'red'$ 
     $v_n = \min(v_n + a, v_r(s))$ 
End

```

(2) Transfer:

$$x_n = x_n + v_n$$

Here $Flag$ denotes the signaling color in front of train n and a denotes the acceleration of trains.

CASE II: the train n is behind a station.

(1) Update speed:

```

If  $gap > x_c$ 
     $v_n = \min(v_n + a, v_{max})$ 
Elseif  $gap < x_c$ 
     $v_n = \max(v_n - b, 0)$ 
Else
     $v_n = v_n$ 
End
 $v_n = \min(v_n, gap)$ 

```

(2) Transfer:

$$x_n = x_n + v_n$$

Here gap is the distance between the train n and the station, and x_c is the distance that train n can enter the station by deceleration.

STEP 2: update the signaling color

CASE I: block k does not contain the station

```

If  $B(k) == 1$ 
    Flag ( $k$ ) = 'red'
Elseif  $B(k + 1) == 1$ 
    Flag ( $k$ ) = 'yellow'
Else
    Flag ( $k$ ) = 'green'
End

```

CASE II: block k contains the station

```

If  $B(k) == 1$ 
    Flag ( $k$ ) = 'red'
Else
    Flag ( $k$ ) = 'green'
End

```

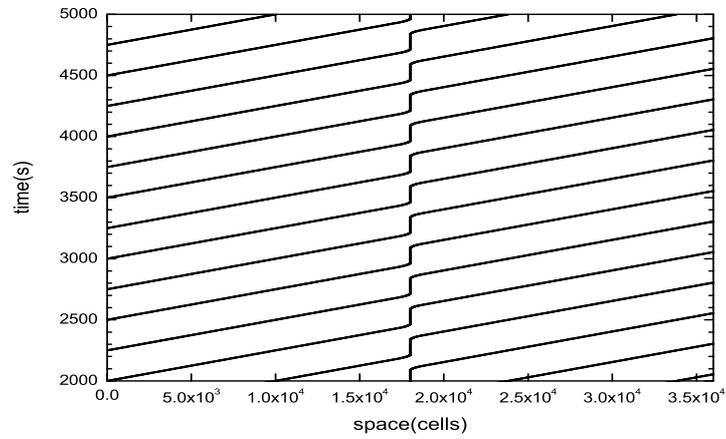
In our method, the boundary condition for CA model is open. It is defined as: (1) when the system updates every I_k times and the first signaling color is green, a train with the velocity $v = v_{max}$ is created at site $i = 1$. Then this train immediately moves according to the update rules. Here I_k is train's time interval. We can set the value of I_k consulting the train's minimal headway T_{min} which can be calculated using the method mentioned by Ding [2]. (2) At site $i = L$, the trains simply move out of the system. In order to compare simulation results to field measurements, one cellular automaton iteration roughly corresponds to 1 sec, and the length of a cell is about 1 m. This means, for example, that $v_{max} = 40\text{cells}/\text{update}$ corresponds to $v_{max} = 144\text{km}/\text{h}$.

In this model, each train runs according to the signaling color in front of it. In the model proposed in [6], however, each train runs according to the position of its front train.

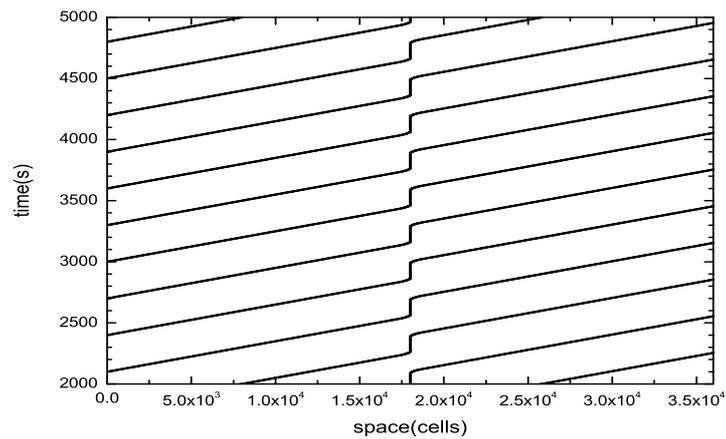
3 Simulation and discussions

We use the CA model proposed in this paper to simulate the railway traffic. One station is designated at the middle of the line. A system of $L = 36000$ cells is considered, and the length of evolution time is $T = 5000\text{s}$. Moreover, $BL = 1200$ cells, $v_{max} = 40\text{cells}/\text{s}$, $v_l = 20\text{cells}/\text{s}$, $a = b = 1\text{cells}/\text{s}^2$, $L_T = 200$ cells, and $T_d = 120\text{s}$. Here L_T denotes the length of the train and T_d denotes the time the train should dwell in the station. Based on these data, we can get the $T_{min} = 250\text{s}$.

First, let us investigate the time-space diagram. Fig. 3(a) shows the space-time diagram of traffic flow when T_d is 120s and I_k equals to T_{min} : 250s. Here we plot 36000 sites in 3000 consecutive time steps. The horizontal direction indicates the direction in which trains move ahead, and the vertical direction indicates time. The positions of the trains are indicated by dots. From Fig. 3(a), we can see that the traffic flow is almost free; the short vertical lines in the middle of the figure denote that trains dwell for 120s in the station. If we let I_k equals 300s, the space-time diagram can be seen in Fig. 3(b). It is clear that



(a) $I_k = 250s, T_d = 120s$



(b) $I_k = 300s, T_d = 120s$

Figure 3: Space-time diagrams.

the density of trains is lower in Fig. 3(b). The reason is that the distance between trains is enhanced with I_k increasing.

Next we focus on the relation between the position and velocity of a train. In Fig. 4, a train departs from site $i = 1$ at the time 2501s. Here the solid line denotes the velocity shift when the dwell time T_d is 120s, and the dotted line denotes the velocity shift when we let T_d equal to 150s. As shown in the figure, when T_d is 120s, this train's velocity keeps the maximum value in the line apart from the situations that the train is close to

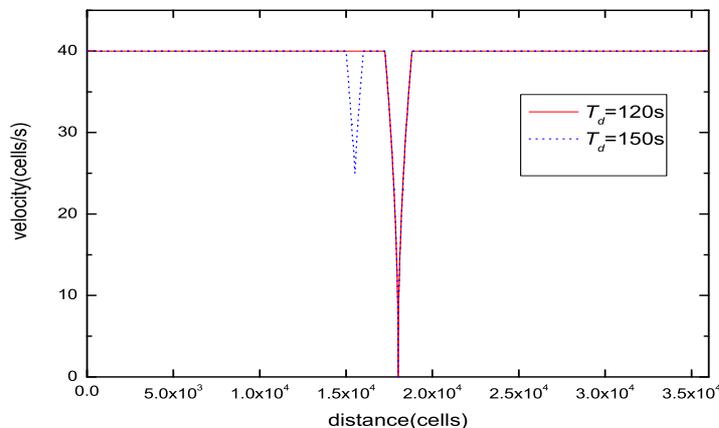


Figure 4: Distance-velocity diagram: $I_k = 250s$.

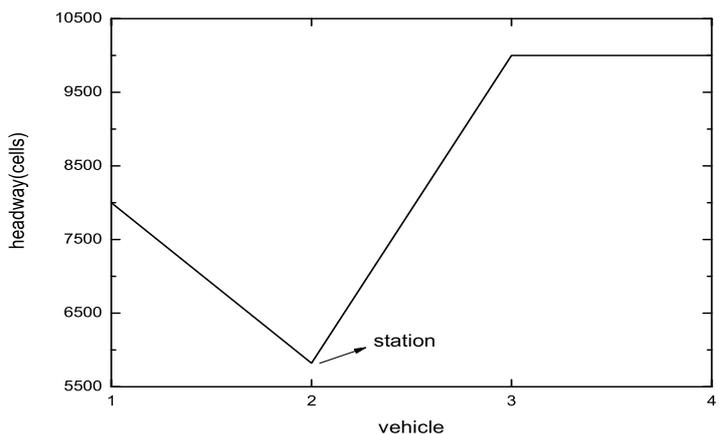


Figure 5: Train-headway diagram: $I_k = 250s$ and $T_d = 120s$.

the station or departs from the station. When T_d is 150s, with train encountering the yellow signaling in a distance behind the station, the train's velocity first decreases to 25 and then accelerates to 40 gradually. In this situation, by increasing the value of I_k , the train possibly keeps running under a green signal at any time.

In real railway traffic, the distance-headway between two trains varies with time. In order to investigate this character of the railway traffic, we measure the distribution of the distance-headway on the railway. $\{h_i\}, i = 1, \dots, M$, is recorded as the distance-headway at a given time t , where h_i is the distance from the train i to the train $i + 1$. Fig. 5 shows the distribution of the distance-headway at the time $t = 5000s$. From this figure, one can

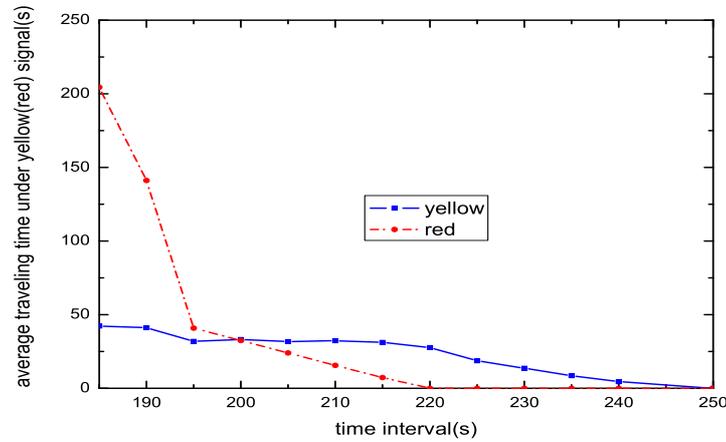


Figure 6: I_k - $Time_y$ ($Time_r$) diagram: $T_d = 120s$.

see that the headway far away from the station is larger. However, the headway near the station is smaller, about 5800 cells. This distribution is largely in accord with the actual situation.

Finally we discuss the I_k 's impact on train running. In Fig. 6, the solid line denotes trains' average running time under a yellow signal, and the dotted line denotes trains' average running time under a red signal. For the sake of convenience, we recorded them as $Time_y$ and $Time_r$, with

$$Time_y = \sum_{i=1}^n y_i/n, \quad Time_r = \sum_{i=1}^n r_i/n,$$

where n is the train number created by the system in 5000s; y_i denotes the time when the train i runs under a yellow signal, r_i denotes the time when the train i runs under a red signal. As shown in this figure, if I_k is larger than 250s then $Time_y$ and $Time_r$ tend to zero. When I_k is less than 250s, $Time_y$ increases as I_k decreases. If I_k decreases to 220s, some trains encounter the red signal. When I_k is less than 195s, the slope of the dotted line is large and the trains' running time under a red signal increases largely. Consequently, the train may run under a yellow signal or a red signal, or stop in front of the red signal if I_k is less than some critical values.

4 Conclusion

In conclusion, we propose a CA model to simulate the three-aspect fixed autoblock system. The simulation results demonstrate that with our model some actual phenomena in railway traffic can be observed. In our model, however, some factors that may affect the traffic

traits have not been taken into account, such as the train types. Some basic assumptions are too ideal. Therefore, more efforts will be made in our future work to study the railway traffic problem via the CA model under more realistic assumptions.

In practice, for some new types of trains, both their acceleration and deceleration have almost run up to 1 m/s^2 . Consequently, in our simulation, trains' acceleration and deceleration are both assumed to be 1 cells/s^2 . It is believed that our assumptions have little influence on the simulation results.

Acknowledgments

This work is financially supported by the National Natural Science Foundation of China through Grant No. 70471088 and the National Outstanding Young Investigator of the National Natural Science Foundation of China through Grant No. 70225005.

References

- [1] D. Chowdhury, L. Santen, A. Schadschneider, Phys. Rep. 329 (2000) 199.
- [2] Y. Ding, Ph.D. Thesis, School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China, 2004.
- [3] M. Fukui, Y. Ishibashi, J. Phys. Soc. Japan 65 1868 (1996).
- [4] M. Grimm, B. Sandblad, Computers in Railways VII, WIT Press, Southampton, 2000, p. 871.
- [5] J.D. Lee, J.H. Lee, C.H. Cho, P.G. Jeong, K.H. Kim, Y.J. Kim, Computers in Railways VII, WIT Press, Southampton, 2000, p. 833.
- [6] K.P. Li, Z.Y. Gao, B. Ning, Int. J. Mod. Phys. C 16 issue 11 in press.
- [7] K. Nagel, Ph.D. Thesis, School of Computer Science, University of Cologne, Cologne, Germany, 1995.
- [8] K. Nagel, Phys. Rev. E 53 (1996) 4655.
- [9] K. Nagel and M. Schreckenberg, J. Phys. I France 2 (1992) 2221.
- [10] D.E. Wolf, Physica A 263 (1999) 438.
- [11] S. Wolfram, Theory and Applications of Cellular Automata, World Scientific, Singapore, 1986.