COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 5, No. 5, pp. 1030-1044

Commun. Comput. Phys.
May 2009

Does a Gravitational Aberration Contribute to the Accelerated Expansion of the Universe?

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Received 3 March 2008; Accepted (in revised version) 1 October 2008

Available online 20 October 2008

Abstract. An arbitrarily small positive value of the gravitational aberration slightly increases the angular momentum of a two-body and multiple body system. This could potentially contribute to the accelerated expansion of the whole Universe. We present some geometrical, physical, geophysical, heliophysical, climatological, cosmological, and astronomical observational arguments, and also numerical tests to support this conjecture. We found a remarkable coincidence between the Hubble constant and the increasing distance of the Moon from the Earth, that is not only due to tidal forces. Numerical examples illustrating the expansion caused by the gravitational aberration are given. This will be modeled by a nonautonomous system of ordinary differential equations with delay.

PACS: 04.20.-q, 04.25.-g

Key words: Hubble constant, causality, gravitation, Earth, Moon, Mars, Runge-Kutta method.

1 Introduction

Gravitational waves were predicted already by Henri Poincaré. In 1905 he conjectured that their speed is the same as the speed of light c (see [1, p. 1507]), i.e., before the same result was postulated by Albert Einstein. If these speeds differ (cf. [2, 3]), then it would be difficult to identify a source of gravitational waves with its optical counterpart, e.g., during (asymmetric) explosions of supernovae. At present, several large projects (GEO, LIGO, VIRGO, LISA, $\cdots$) are being developed to measure the speed of gravitational waves and determine the direction, from which they come. However, for the time being these waves have not yet been detected.

First of all, we will focus our attention on the following classical geometrical example inspired by Sir A. Eddington (see [4, pp. 94 and 204]). Let A and B be two bodies of

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equal masses. Assume for a moment that their orbits are circular about their common centre of gravity. If A attracts B at its instantaneous position and also B attracts A at its instantaneous position (i.e., the speed of their mutual gravitational interaction is infinite), then by Newtonian mechanics these forces are in the same line and in balance.

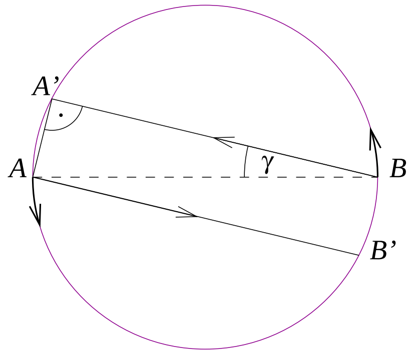

Figure 1: Schematic illustration of two interacting bodies of equal masses. The gravitational aberration angle $\gamma = \angle ABA'$ is extremely small.

On the other hand, suppose now that the speed of their mutual gravitational interaction is finite, i.e., B is attracted by A towards its previous position A' (see Fig. 1). Similarly A is attracted by B in the direction of its previous position B'. Then a couple of nonequilibrium forces arises which acts permanently and thus, increases the angular momentum and total energy of this system (see [5]). By the Thales theorem the triangle $AA'B$ is right and

$$|A'B| < |AB|. \tag{1.1}$$

Hence, the attractive forces (in this postnewtonian mechanics) are sightly larger than if they would act along the hypotenuse AB.

Let us point out that Fig. 1 is slightly imprecise. Since (1.1) is valid, the attractive force is larger than that from the Newtonian theory. Consequently, an arbitrarily small positive value of the gravitational aberration γ of the considered binary system increases not only its angular momentum, but also prolongs the orbital period. Thus, the corresponding trajectories constitute two very slowly expanding spirals (see Fig. 2).

The value $\gamma \leq 0$ evidently contradicts to causality. For instance, if one of the bodies (asymmetrically) explodes, then the second body has to orbit for some time along the unchanged trajectory, because the speed of gravitation is finite and the associated gravitational fields need a time interval of positive length to change.

In 2000, Steven Carlip [6] showed that in general relativity the gravitational aberration is almost cancelled out up to the order v^3/c^3 by velocity-dependent interactions, where v is the speed of an observed object. Thus, the real value of gravitational aberration is probably also much smaller than the aberration of light v/c. Due to this property the orbit of two bodies is seemingly very stable. Also sunrays arriving at the Earth are not parallel with the vector of the attractive gravitational force of the Sun. How to interpret

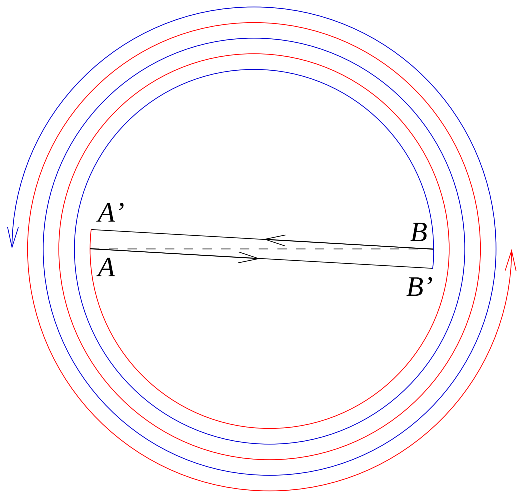

Figure 2: Trajectories corresponding to two interacting bodies of equal masses constitute a double spiral. In this case, inequality (1.1) holds as well.

such a paradoxical phenomenon and find that it does not contradict the general relativity is discussed in [3, 6, 7].

Slowly expanding spiral trajectories are observed also numerically for a general n-body problem with delays due to the finite speed of gravitational interaction (see Section 5). Thus, an arbitrarily small positive value of gravitational aberration contributes to an expansion of any binary or multiple body system and it should be taken into account when dealing with the expansion of the whole Universe (see [5]). In this paper, we present other geophysical, heliophysical, climatological, cosmological, computational, and astronomical observational arguments supporting this conjecture. We shall consider only small nonrelativistic speeds and weak gravitational fields. We shall see that gravitational aberration effects are visible on small and also large time and space scales.

Finally note that there exist close binary pulsars whose orbits do not expand with time, but decay. In this case, strong magnetic and gravitational fields are present, the system loses energy due to electromagnetic and gravitational waves, and these effects are much stronger than very weak effects coming from the gravitational aberration.

2 A remarkable coincidence

By laser retroreflectors installed on the Moon by Apollo 11, 14, 15, and Lunokhod 2, we know that the mean distance

$$D = 384400\,\mathrm{km} \tag{2.1}$$

between the Earth and the Moon increases about

$$\Delta = 3.84\,\mathrm{cm} \quad \text{per year}. \tag{2.2}$$

Hence, the mean observed secular increase of D is given by

$$\left(\frac{dD}{dt}\right)_{observed} = \frac{\Delta}{T} = 1.2\times 10^{-9}\mathrm{m/s}, \tag{2.3}$$

where $T = 31558149.54$ s (=365.25636 days) is the sidereal year.

It is said that this value is mainly due to tidal forces caused by the Moon and the Sun. Nevertheless, in Section 3 we show that only cca $55-60\,\%$ of Δ can be explained by these forces.

In Section 1 we declared that the finite speed of gravitational interaction contributes not only to the value Δ, but also to the expansion of the whole Universe. This expansion is given by the Hubble constant

$$H_0 = 20\,\mathrm{km s^{-1}(Mly)^{-1}}.$$

Note that $H_0 = c/(15\mathrm{Gyr})$, where 15 Gyr is an approximate value of the age of Universe 13.7 Gyr.

Now let us relate the value of the Hubble constant to the distance D given by (2.1) during one year,

$$\begin{aligned} H_0 &= 20\,\mathrm{km s^{-1}(Mly)^{-1}} = 2\,\mathrm{cm s^{-1} ly^{-1}} = \frac{2}{c}\,\mathrm{cm s^{-1} yr^{-1}} \\ &= \frac{2}{0.78\times D}\,\mathrm{cm yr^{-1}} = 2.56\,\mathrm{cm yr^{-1}} D^{-1}. \end{aligned} \tag{2.4}$$

We observe that this value is surprisingly very close to the measured value Δ in (2.2), i.e., the speed of expansion of the Universe is very similar and thus comparable with the mean receding speed of the Moon from the Earth. We do not know yet, what portion of H_0 is due to the initial explosion (Big Bang) and subsequent gravitational interaction that slows down the expansion, and what portion is due to the gravitational aberration and other effects. In the sequel, we try to estimate these relative contributions.

During the last 7×10^9 years the expansion of the Universe accelerates. This observation is based on measurements of the satellite WMAP and the fact that the luminosity of very distant supernovae of type Ia is up to 15 % smaller than it should be (see [8]). It is said that the observed acceleration is due to dark energy, whose nature is unknown. However, gravitational aberration, which also has a repulsive character, could be an alternative candidate to explain this acceleration. From Section 1 we know that an arbitrarily small positive value of the gravitational aberration slightly (but permanently) increases the angular momentum of a two-body or n-body system, and also its potential and total energy. Such a system can be formed by planets, stars, galaxies, clusters of galaxies, etc. This could eventually lead to the accelerated expansion of the whole Universe.

An idealized time behavior of the Hubble constant $H = H(t)$ is depicted in Fig. 3. By (2.4) its current value is approximately equal to $H(14\times 10^9) \doteq H_0$. The distance of two sufficiently distant galaxies (in simple cosmological models) increases as $t^{2/3}$, provided

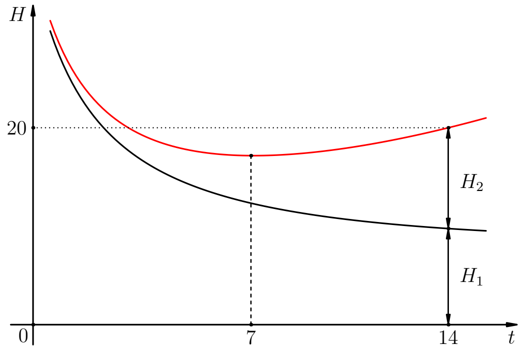

Figure 3: Time behavior of the Hubble constant. All data are only approximate. The time t is given in Gyr.

the speed of expansion is driven only by gravitational forces (see [9, p. 735]). Hence, the receding speed of the considered galaxies behaves like $t^{-1/3}$ and the corresponding part of the Hubble constant is of the form $H_1(t) = Ct^{-1/3}/t^{2/3} = Ct^{-1}$, where $C > 0$ is a multiplicative constant. Set $H_2(t) := H(t) - H_1(t)$, i.e.,

$$H = H_1 + H_2, \tag{2.5}$$

where H_1 corresponds to the gravitational interaction that slows down the expansion and H_2 is the remaining part of H that is not due to attractive gravitational forces and that increases with time. Both the parts H_1 and H_2 have an "averaged" character, i.e., all local irregularities are ignored.

Gravitational aberration thus contributes only to the part H_2 of the entire value of the Hubble constant. The model proposed in this paper explains, from where we permanently get the energy necessary for an accelerated expansion of the Universe. Moreover, the increasing character of the function H_2 shows why the slowing expansion turned into an accelerating one.

3 Influence of a deceleration of the Earth's rotation to Δ

Let us estimate the contribution of tidal friction to the value Δ (see (2.2)). Consider the binary system Earth-Moon with masses

$$m_1 = 5.976 \times 10^{24}\,\text{kg}, \quad m_2 = 7.350 \times 10^{22}\,\text{kg}, \tag{3.1}$$

and assume, for simplicity, that their orbits are circular. Then the corresponding distance (see (2.1)) can be expressed as

$$D = R_1 + R_2, \tag{3.2}$$

where

$$R_1 = \frac{Dm_2}{m_1 + m_2} \qquad \text{and} \qquad R_2 = \frac{Dm_1}{m_1 + m_2} \tag{3.3}$$

are distances of the Earth and the Moon from the Newtonian centre of gravity, respectively.

By the conservation of the total momentum of this system, the value

$$M = I_1\omega_1 + I_2\omega_2 + m_1R_1v_1 + m_2R_2v_2 \tag{3.4}$$

has to be constant. Here v_1 and v_2 are the speeds of the Earth and Moon, respectively, relative to their centre of gravity,

$$I_1 = 8.036 \pm 0.008 \times 10^{37}\,\mathrm{kgm}^2 \tag{3.5}$$

is the inertia moment of the Earth (see [10]), $\omega_1 = 2\pi/T_1 = 7.292 \times 10^{-5}\mathrm{s}^{-1}$ is the angular frequency of the Earth, $T_1 = 86164.1$ s is the sidereal day,

$$\omega_2 = \frac{2\pi}{T_2} = 2.669 \times 10^{-6}\mathrm{s}^{-1} \tag{3.6}$$

is the angular frequency of the Moon, and $T_2 = 27.322T_1$. Moreover, using the formula for the moment of inertia of a homogeneous ball [11, p. 109], we find for the inertia moment of the Moon (whose density ρ increases towards its center) that

$$I_2 < \frac{8}{15}\pi r_2^5 \times \rho_2 = 8.849 \times 10^{34}\ \mathrm{kgm}^2, \tag{3.7}$$

where $\rho_2 = 3340\ \mathrm{kg/m}^3$ denotes the mean density of the Moon and $r_2 = 1737$ km is its radius. (Note that the term $I_2\omega_2 < 2.36 \times 10^{29}\ \mathrm{kg\,m^2s^{-1}}$ corresponding to the Moon is much smaller than $I_1\omega_1 = 5.86 \times 10^{33}\ \mathrm{kgm^2s^{-1}}$, but we have to compare their time derivatives.)

According to [12], the Earth's rotation slows down mainly due to tidal forces of the Moon (cca 68.5 %), but also of the Sun (cca 31.5 %). It is also known that the length of a day increases by $\tau_1 = 1.7 \times 10^{-5}$s per year during the last 2700 years (see, e.g., [13, p. 270], [14, p. 62]). Therefore, the increase $\tau = 0.685\tau_1$ corresponds to the Moon and $0.315\tau_1$ to the Sun. Let us set

$$\overline{\omega}_1 = \frac{2\pi}{T_1 + \tau}.$$

We see by (3.5) that the decrease of the Earth's angular momentum caused by the tidal forces of the Moon is

$$\frac{d\omega_1}{dt} = \frac{\overline{\omega}_1 - \omega_1}{T} = -\frac{2\pi}{T}\frac{\tau}{T_1(T_1+\tau)} = -3.123 \times 10^{-22}\mathrm{s}^{-2},$$

i.e.,

$$I_1\frac{d\omega_1}{dt} = -2.509 \times 10^{16}\ \mathrm{kgm^2s^{-2}}, \tag{3.8}$$

where T is the sidereal year.

The Moon also reduces its angular momentum due to (2.2) and the 1:1 resonance between the orbital period T_2 and the Moon's rotation. Since $m_2 \ll m_1$, we can apply

Kepler's third law which states that D^3/T_2^2 is constant. Hence, the product $\omega_2^2 D^3$ is by (3.6) also constant. Differentiating $\omega_2^2 D^3$ with respect to time, we obtain

$$2\omega_2 \frac{d\omega_2}{dt} D^3 + 3\omega_2^2 D^2 \frac{dD}{dt} = 0,$$

i.e.,

$$\frac{d\omega_2}{dt} = -\frac{3}{2}\frac{\omega_2}{D}\frac{dD}{dt}. \tag{3.9}$$

From this, (2.1), (2.3), (3.6), and (3.7) we get

$$\left| I_2 \frac{d\omega_2}{dt} \right| < 1.1 \times 10^8 \ \mathrm{kgm^2 s^{-2}},$$

i.e., the decrease of the Moon's angular momentum is negligible with respect to the value given in (3.8). Therefore, the decrease of the rotational momentum in (3.8) must be compensated by the increase of the orbital momentum $m_1 R_1 v_1 + m_2 R_2 v_2$ in (3.4). Since the angular frequency of the Moon is the same as the angular speed of the Earth about their common center of gravity, we have $\omega_2 = v_2/R_2 = v_1/R_1$. By the momentum conservation law $m_1 v_1 = m_2 v_2$, (3.2), and (3.3) we find that

$$\begin{aligned} m_1 R_1 v_1 + m_2 R_2 v_2 &= (R_1 + R_2) m_1 v_1 = D m_1 v_1 \\ &= D m_1 R_1 \omega_2 = D^2 \frac{m_1 m_2}{m_1 + m_2} \omega_2. \end{aligned}$$

From this and (3.9), we get by differentiating (3.4) with respect to time that

$$I_1 \frac{d\omega_1}{dt} = -\frac{m_1 m_2}{m_1 + m_2}\Big(\frac{d\omega_2}{dt} D^2 + 2\omega_2 D \frac{dD}{dt} \Big) = -\frac{m_1 m_2}{m_1 + m_2} \frac{\omega_2 D}{2} \frac{dD}{dt}.$$

Substituting from (2.1), (3.1), (3.6), and (3.8) yields

$$\frac{dD}{dt} = 0.674 \times 10^{-9} \mathrm{m/s}, \tag{3.10}$$

which is only 56% of the observed value in (2.3). To explain this discrepancy, Novotný [14] considers a time dependent angular momentum $I_1 = I_1(t)$ which yields that $-dI_1/dt$ is of order $10^{20} - 10^{21}$ kg m^2/s. However, this would require that a large transport of mass toward the Earth's center must exist for at least 2700 years.

We are, of course, not able to take into account all nongravitational forces that have an influence on Δ such as the solar wind, thermal radiation of the Earth and Moon, the Yarkovsky effect, collision with interplanetary dust and meteorites, presence of magnetic fields, etc. Nevertheless, their effect is almost negligible when compared with (3.10). Also the gravitational influence of the other bodies of the Solar system, gravitational radiation, etc., have only a very small influence on the measured value Δ.

Geophysicists admit (see, e.g., [14, p. 67]) that it is not easy to explain the large discrepancy between the observed value in (2.3) and the value derived from the tidal forces (3.10). The distance between the Earth and the Moon should increase by (3.10) about

$$\Delta_{\text{tidal}} = \frac{dD}{dt} T \approx 2.13\,\text{cm} \quad \text{per year.} \tag{3.11}$$

We observe that the difference

$$\Delta_{\text{nontidal}} \approx 1.71\,\text{cm} \quad \text{per year} \tag{3.12}$$

between the measured value Δ in (2.2) and the value (3.11) corresponding to tidal forces is slightly less than H_0 given by (2.4). However, the gravitational aberration contributes only to the part H_2 of H_0 (see (2.5)). This enables us to state a hypothesis:

The gravitational aberration caused by the finite speed of gravity not only contributes to the increase of the distance Earth-Moon, but also to the expansion of the whole Universe.

4 Further arguments

Denote by $E(t_1)$ the energy of an isolated system of space objects at time t_1 and by $E(t_2)$ its energy one year later. From the energy conservation law, we have

$$\left| \frac{E(t_2)}{E(t_1)} - 1 \right| = \varepsilon,$$

for $\varepsilon = 0$. However, what would happen if the gravitational aberration, caused by the finite speed of gravitational interaction, would give, for example, that $\varepsilon \approx 10^{-10}$? Then it would be very difficult to detect this weak energy source. For instance, the energy required to move the Moon 1 cm away from the Earth equals 10^{18} J, whereas the kinetic energy of the Moon itself is 3.7×10^{28} J. Therefore, it is necessary to consider very long time scales to show that $\varepsilon > 0$. We will illustrate this by further examples.

4.1 Was the Earth closer to the Sun?

It is known that the radius of Earth's orbit may be at most 5% larger or 2% smaller than 1 AU to guarantee a suitable climate for photosynthesis and the existence of life at the present time. Such a ring is called the *ecosphere*. Shortly after the origin of the solar system (4.5 Gyr ago), the luminosity of the Sun was only 70% of its current value. Since the Sun is a star of the main sequence, by evolution models its luminosity increased approximately linearly (see Fig. 4). Thus, during the origin of life on the Earth 3.5 Gyr ago, the luminosity of Sun was about 77% of its actual value. However, if the Earth were 150 million farther away from the Sun at that time, life could not arise, since the surface of

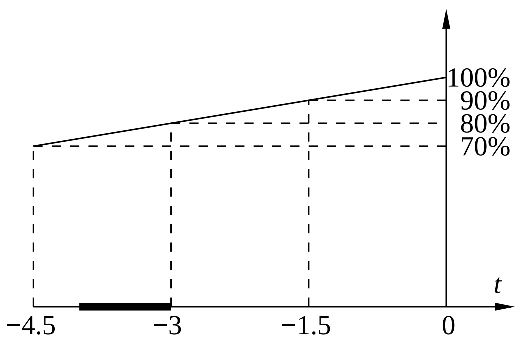

Figure 4: Relative luminosity of the Sun from the origin of the Solar system up to today. The time t is given in Gyr.

the Earth would be frozen, even though there was a different composition of atmosphere, higher radioactivity, vulcanism, etc.

To guarantee a suitable climate for the origin of life, the Earth had to be many millions of kilometers closer to the Sun. If the Earth would be, for instance, 10 million km closer, then the averaged receding speed from the Sun would be almost 3 meters per year. By Kepler's third law we find that the orbital period would change only by less than 1 ms. Unfortunately, such small changes of orbital parameters can be verified neither by direct measurements of the Earth-Sun distance nor from the change of the Earth's orbital period T. Using the fact that the Sun's luminosity increases approximately linearly, we find that the receding speed of the Earth from the Sun

$$\mathbf{v}=\frac{150\times 10^9(1-\sqrt{0.77})}{3.5\times 10^9}\doteq 5.25\,\mathrm{m\,yr}^{-1}$$

during the time period starting from the origin of life up to today corresponds to almost constant energy absorbed by 1 m^2 per second that is equal to the solar constant

$$L=1.36\,\mathrm{kW\,m}^{-2}.$$

For comparison note that the current value of the Hubble constant related to 1 astronomical unit (cf. Fig. 3 and (2.4)) is

$$H_0=10\,\mathrm{m\,yr}^{-1}(\mathrm{AU})^{-1}.$$

Note that tidal forces from the Sun, solar wind, and the decreasing mass of the Sun can explain a receding speed of only less than 1 cm per year.

4.2 Was Mars closer to the Sun?

The current averaged temperature on Mars is $-63°$C. Thus, Mars also must have been much nearer to the Sun to have liquid water on its surface 3-4 Gyr years ago. Its averaged temperature and atmospheric pressure were higher. Otherwise, we could not observe there dried-up channels, especially when the luminosity of Sun was smaller, about 73-80 % of its current value (see Fig. 4). If Mars in the past were to be on the same orbit as at present, then the solar energy related to 1 m^2 per second perpendicular to the sun rays would be approximately 3 times smaller than the solar constant L. Moreover, the albedo of Mars was higher than the current value 0.25, since there were water clouds feeding many rivers (see `http://www.google.com/mars`). Ice and snow were not only at polar ice caps, but also at other regions, which also increased albedo. These indirect arguments show that the receding speed of Mars from the Sun must have been several meters per year. It is perhaps again the gravitational aberration that contributes to the expansion of Mars' trajectory.

4.3 Fast satellites

In the solar system we know of about 20 satellites that are below the corresponding stationary orbit. Their orbital periods around their mother planet are less than the period associated to the planet's rotation along its axis. We will call them *fast satellites.* Mars has such a satellite Phobos (9 378) and Jupiter has Metis (127 960) and Adrastea (128 980); the numbers in parentheses determine their radii in kilometers. Around Uranus there are at least eleven fast satellites: Cordelia (49 752), Ophelia (53 764), Bianca (59 165), Cressida (61 777), Desdemona (62 659), Juliet (64 358), Portia (66 097), Rosalind (69 927), Cupid (74 392), Belinda (75 255), Perdita (76 416), and around Neptune five: Naiad (48 227), Thalassa (50 075), Despina (52 526), Galatea (61 953), and Larissa (73 550).

The tidal bulges continuously reduce potential energy and orbital periods of the satellites. Since the inertia moment is constant (as in (3.4)), their speed slowly increases, and also the planets' rotations slightly speed up during this process. Thus, all these fast satellites approach their planets along spiral trajectories. From a statistical point of view it is very unlikely that all these satellites were captured, since all of them orbit in the same direction in circular trajectories and their inclinations are very small. Consequently, they have been mostly in their orbits approximately 4.6 Gyr (even though they could be parts of larger disintegrating satellites).

Denoting by m the mass of a planet and by T_s its sidereal rotation, the radius of stationary orbit (see Table 1) is given by

$$r_{\rm stac} = \sqrt[3]{\frac{GmT_s^2}{4\pi^2}},$$

where $G = 6.673 \times 10^{-11}$ m^3 kg^{-1}s^{-2} is the gravitational constant.

Tidal forces are proportional to m/r^3, where m is again the mass of a planet and r is the radius of a given satellite orbit. For all the above-mentioned satellites the value m/r^3 is of the same order as for Phobos (for some larger, for some smaller). Moreover, all of them are bigger than Phobos. It can be estimated (like in Section 3) that tidal forces reduce the radius of Phobos' orbit 1.8 cm per year. Assuming a similar speed also for the

Table 1: The mass, sidereal rotation and the radius of stationary orbit.

Planet	$m \times 10^{-24}$ [kg]	T_s [hour]	$r_{\rm stac}$ [km]	No. of fast satellites
Mars	0.642	24.6229	20,427	1
Jupiter	1898.6	9.925	159,988	2
Uranus	86.83	17.24	82,675	11
Neptune	102.43	16.11	83,496	5

other fast satellites, we find that during the last 4.6 Gyr they would be 82 800 km closer to their mother planet. How is it possible that these fast satellites have not fallen down onto their mother planet due to tidal forces? All of them (except for Phobos) stay on relatively high orbits of radii 0.58-0.92 $r_{\rm stac}$. It is perhaps again the gravitational aberration that acts in the opposite direction than the tidal forces and thus protects these satellites against crashing onto their mother planets.

4.4 Does our Galaxy expand as well?

The diameter of our Galaxy is about $d = 10^5$ ly. By (2.4) the current value of the Hubble constant on this distance is

$$H_0 = 2\ \mathrm{kms}^{-1} d^{-1}.$$

Let us make the following very rough estimate. Suppose that our Galaxy expanded from some small protogalaxy by the averaged speed 2 km/s for 13 Gyr. Then its size would be

$$2\,\mathrm{kms}^{-1} \times 13 \cdot 10^9\,\mathrm{yr} \doteq c \times 9 \cdot 10^4\,\mathrm{yr} = 9 \cdot 10^4\,\mathrm{ly},$$

which is in a good agreement with the real diameter d.

Assume to the contrary that galaxies do not change their sizes. Then $10-13$ Gyr ago their density would be much larger. For instance, for the red shift $z=2$ (corresponding roughly to the Hubble Field South), when space was $(z+1)$ times smaller, we should observe a three times higher density of galaxies. Since protogalaxies at that time were smaller, the higher density is not observed.

4.5 Aberration of gravitational waves

The finite speed of light c combined with the speed $v > 0$ of an observer causes a positive light aberration angle $\alpha \doteq v/c$ in the observed position of a star (see Fig. 5). Consider now the following situation. Suppose that the star explodes (asymmetrically) and that electromagnetic and gravitational waves have exactly the same speed of propagation as the general theory of relativity states. Assume further that the telescope in Fig. 5 is replaced by an instrument that can detect the direction from which the gravitational waves come from. They will come from the same direction as light. However, the gravitational aberration angle γ corresponding to the gravitational force will be much smaller than the

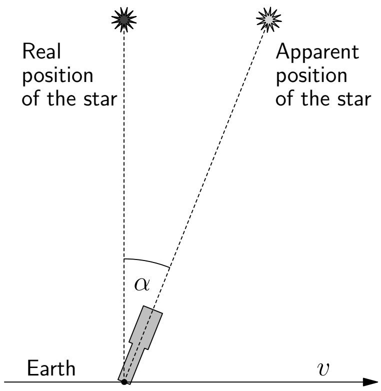

Figure 5: The light aberration angle α appears due to the movement of an observer.

aberration of gravitational waves α (cf. [6]). The Newtonian theory assumes that $\gamma=0$, but causality dictates that $\gamma>0$.

5 Numerical tests with gravitational aberration

Now let us verify our conjecture on expansion numerically. Consider two mass points m_1 and m_2 in the two- or three-dimensional space equipped with the Euclidean norm $|\cdot|$. Denote by c_g the real "Newtonian" speed of gravity. Introducing a delay into gravitational interactions, the classical autonomous Newtonian system of ordinary differential equations can be rewritten as the following nonautonomous system for two trajectories r_1 and r_2:

$$r_1''(t)=G\frac{m_2(r_2(t-d_2(t))-r_1(t))}{|r_2(t-d_2(t))-r_1(t)|^3}, \tag{5.1}$$

$$r_2''(t)=G\frac{m_1(r_1(t-d_1(t))-r_2(t))}{|r_1(t-d_1(t))-r_2(t)|^3} \tag{5.2}$$

with two variable delays d_1 and d_2 satisfying (5.4). Consider the initial conditions:

$$r_i(t)=p_i(t), \quad r_i'(t)=v_i(t), \quad t\in[\tau,0], \quad i=1,2, \tag{5.3}$$

where $\tau\leq 0$ is an appropriate given number and p_i and v_i are given functions characterizing previous positions and velocities. This postnewtonian model does not take into account gravitational waves, but it involves the gravitational aberration.

If $c_g=\infty$ then $\tau=d_1=d_2=\gamma_1=\gamma_2=0$ and system (5.1)-(5.2) reduces to the classical Newton two-body problem. For $c_g<\infty$ the delay functions satisfy the relations (cf. Fig. 6)

$$d_1(t)=\frac{|r_1(t-d_1(t))-r_2(t)|}{c_g}, \quad d_2(t)=\frac{|r_2(t-d_2(t))-r_1(t)|}{c_g}, \tag{5.4}$$

i.e., each d_i has to be calculated iteratively using the classical Banach fixed-point theorem. For the theory of ordinary delay differential equations we refer to [16, 17, 18].

Assume now that

$$m_1 R_1 = m_2 R_2, \tag{5.5}$$

where R_1 and R_2 are distances from the Newtonian centre of gravity. Let us define

$$p_1 = (R_1,0), \quad p_2 = (-R_2,0), \quad v_1 = \Big(0, \frac{\sqrt{Gm_2R_1}}{R_1+R_2}\Big), \quad v_2 = \Big(0, -\frac{\sqrt{Gm_1R_2}}{R_1+R_2}\Big).$$

These values yield exactly circular orbits for $\tau = 0$ in (5.3) and $c_g = \infty$. They are employed to establish initial conditions (5.3) for $c_g < \infty$. This, of course, requires to store old values of r_1 and r_2 throughout the computation due to initial conditions (5.3). *A big advantage of computer simulations is that we can easily perform many tests for various parameters appearing in* (5.1)-(5.4).

Example 5.1. If $m_1 > m_2 = 0$, the second body orbits around the first one in its stationary gravitational field. In this case, no expansion of the trajectory of the second body was obtained (neither analytically, nor numerically). Its orbit is perfectly circular. Indeed, if $m_2 = 0$, then by (5.1) and (5.5), we get

$$r_1'' = 0, \quad R_1 = r_1 = v_1 = 0, \quad r_2'' = -Gm_1 r_2 / |r_2|^3, \quad \gamma = 0,$$

i.e., no positive gravitational aberration appears.

Example 5.2. The analytical solution of problem (5.1)-(5.4) is not known for $c_g < \infty$. Numerically calculated trajectories r_1 and r_2 for $m_1 = m_2 > 0$ and $c_g \le c$ are depicted in Fig. 2. We see that they are quite unrealistic, since they form two quickly expanding spirals, which does not correspond to astronomical observations. However, model (5.1)-(5.4) yields quite satisfactory results for $c_g \gg c$.

Example 5.3. The largest value of gravitational aberration is obtained when $m_1 \approx m_2$ (cf. Fig. 1 and 2). However, in the Solar system such objects do not exist. For the Earth and Moon the ratio $m_1 : m_2$ equals $81:1$. So let us again consider this close binary system Earth-Moon with masses given by (3.1) and the corresponding distance (2.1) and (3.2). To get the receding speed derived in (3.12), we have to take $c_g = 4.287 \times 10^{15}$ m/s for the considered postnewtonian model. In this case the gravitational aberration angle at the point B representing the Moon in Fig. 6 is

$$\gamma = \frac{v}{c_g} \doteq 0.5 \times 10^{-7} \ \text{arc seconds},$$

where $v = r_2' \doteq 1$ km/s, and the two trajectories form two very slowly expanding spirals. Note that the light aberration angle of the Moon is $\alpha = v/c = 0.7''$ and of the Earth (observed from the Moon) is 81 times smaller.

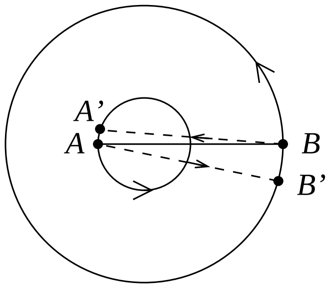

Figure 6: Illustration of gravitational interaction between two bodies of unequal masses $m_1 > m_2$. If $m_2 \to 0$ then the aberration angle $\angle ABA'$ vanishes.

Example 5.4. We observed numerically expanding trajectories for three bodies of equal masses that are at the vertices of an equilateral triangle and that orbit at the same speed about their centre of gravity. A similar phenomenon was achieved for the case marked in Fig. 2, where the third body lies at the midpoint of AB. Expanding trajectories were also obtained for a system consisting of two double stars of equal masses.

All calculations were done in extended 10 byte precision by the standard fourth order explicit Runge-Kutta method (see [19]) which gives a surprisingly small discretization error when the orbits are circular [5, 20]. For instance, the time step $\Delta t = 100$s, that produces practically the same results as $\Delta t/2$, yields for $c_g = \infty$ a total numerical error after 1000 revolutions of only 17 mm which is almost negligible.

6 Conclusions

We showed that the energy needed for the accelerated expansion of the Universe may come from a finite speed of gravitational interaction that causes a positive value of gravitational aberration. Although this value is much smaller than the aberration of light, it makes "elliptic" trajectories of two bodies very slowly expanding, in general. This phenomenon is observed analytically, numerically, and has a lot of consequences. For instance, it explains a relatively large receding speed of the Moon from the Earth that cannot be explained by tidal forces. Also the magnitude of the receding speed of the Earth from the Sun seems to be just right for an almost constant influx of solar energy during the last 3.5 Gyr.

Acknowledgments

This paper was supported by Institutional Research Plan nr. AV0Z 10190503 and Grant nr. IAA 100190803 of the Academy of Sciences of the Czech Republic. The author is indebted to Jan Brandts (Amsterdam), Lawrence Somer (Washington, D.C.), Miroslav

Brož, Filip Křížek, Ctirad Matyska, Jiří Podolský, and Vojtěch Pravda from Prague for fruitful discussions.

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