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ON BASIC SEMICONDUCTOR EQUATIONS WITH HEAT CONDUCTION*

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Abstract We prove the global existence of solution to basic semiconductor equations with heat conduction; If the domain is narrow in one direction, then the basic equations has a unique steady-state which is locally asymptotically stable.

Key Words Super-subsolution; fixed point theorem; L^p estimate and Schauder estimate.

Classification 35K57, 35M10.

1. Introduction

(1) We consider a nonlinear system of partial differential equations arising from semiconductor theory (see [1]):

$$\begin{cases} \Delta\psi = \dfrac{q}{\varepsilon}(n - p - N(x)) & (1.1) \\ \operatorname{div}\vec{J}_n - \dfrac{\partial n}{\partial t} = R(n,p,\theta) & (1.2) \\ \operatorname{div}\vec{J}_p - \dfrac{\partial p}{\partial t} = R(n,p,\theta) & (1.3) \\ k\dfrac{\partial \theta}{\partial t} - \Delta\theta = H(n,p,\psi,\nabla n,\nabla p,\nabla\psi) & (1.4) \end{cases}$$

where ψ is the electrostatic potential, n and p are the densities of mobile holes and electrons respectively, θ is the temperature. $\vec{J}_n = D_n\nabla n - \mu_n n\nabla\psi$, $\vec{J}_p = D_p\nabla p + \mu_p p\nabla\psi$ are the hole and electron current densities, D_n and D_p are the diffusion coefficients for holes and electrons, μ_n and μ_p are the mobility of holes and electrons. R is the net recombination rate. $N(x)$ is the density of ionized impurities. ε and q denote the dielectric permittivity and the unit change. k is associated with the material. We assume $\mu_n, \mu_p, D_n, D_p, \varepsilon, q, k$ are positive constants. $R(n,p,\theta) = r(n,p,\theta)(np - l(\theta))$. H represents local produced heat, one of the simplest forms is $-(\vec{J}_n + \vec{J}_p)\nabla\psi$.

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The system (1.1)–(1.4) governs the transport of mobile carriers in a semiconductor device. For details please see [2] or [3]. Most researchers neglect the influence of change of temperature. Strictly speaking, the heat equation (1.4) should be included.

(2) On boundary condition: In this article we will only consider Dirichlet boundary condition.

(3) Known results and recent developments: there are many results when θ is considered as a constant. On steady-states, the first existence result is established in [4] under condition $R = 0$; for more general existence result of steady-states, we refer to [5–7]. On uniqueness of steady-states, we refer to [8–11] where partial results are included. Generally physical considerations show that one has to expect non-uniqueness of steady-states. On the global existence and uniqueness of solutions of (1.1)–(1.3) (θ is regarded as a constant), see [12], [13], [1]. On asymptotic behavior of solutions of (1.1)–(1.3), partial results are obtained in [14] under very special boundary conditions. On the basic equations with heat conduction, as far as we know, only Seidmann [15] and Seidmann and Troianiello [16] obtained some results. In [15] they proved the existence and uniqueness of solutions of (1.1)–(1.4) and the existence of periodic solutions. In [16] they showed some results on the existence and uniqueness of solutions to (1.1)–(1.4) and the existence of steady-state. No result is known on the asymptotic behavior of the solutions of (1.1)–(1.4).

Recently we learned following interesting results from [17]: if Ω is sufficiently narrow in one direction, θ is a constant, then (1.1)–(1.3) has a unique steady-state and the solutions of (1.1)–(1.3) converge to the unique steady-state exponentially.

(4) Our main results: in this paper we will try to extend the results of [17] to system (1.1)–(1.4): first we established the global existence of solutions of (1.1)–(1.4); if Ω is sufficiently narrow in one direction, then (1.1)–(1.4) has a unique steady-state and it is locally asymptotically stable.

2. Existence and Uniqueness of Solutions of (1.1)–(1.4)

We impose following initial and boundary conditions:

$$\begin{aligned} &n,p,\theta|_{t=0} = n_0(x), p_0(x), \theta_0(x) \\ &n,p,\theta,\psi|_{\partial\Omega} = \bar{n}(x,t), \bar{p}(x,t), \bar{\theta}(x,t), \bar{\psi}(x,t) \end{aligned} \tag{2.1}$$

Theorem 1 *Under following conditions, the system (1.1)–(1.4), (2.1) has a unique solution* $(n,p,\psi,\theta) \in [C^{2+\alpha,1+\alpha/2}(Q_T)]^4$ *for* $T > 0 : 0 \le r(n,p,\theta) \le r_1$, $0 \le l \le l_1$, $(0,0) \le (\bar{n},\bar{p})$, $(n_0,p_0) \le (1,1)$, $0 \le N(x) \le \bar{N}$, *where* r, l, H *are Lipschitz continuous,* $r_1, l_1, \bar{N}$ *are positive constants,* $\bar{n},\bar{p},\bar{\theta},\bar{\psi} \in C^{2+\alpha,1+\alpha/2}(Q_T)$, $n_0,p_0,\theta_0 \in C^{2+\alpha}(\bar{\Omega})$, $N(x) \in C^{\alpha}(\bar{\Omega})$, $|H(n,p,\psi,\nabla n,\nabla p,\nabla\psi)| \le H_0(n,p,\psi,\nabla\psi) + H_1(n,p,\psi,\nabla\psi)|\nabla n|^{l_0} + H_2(n,p,\psi,\nabla\psi)|\nabla p|^{l_0}$, *where* H_1, H_2 *are continuous,* l_0 *is some positive constant, and the compatibility conditions on* n_0 *and* $\bar{n}$, p_0 *and* $\bar{p}$, θ_0 *and* $\bar{\theta}$ *are always assumed.*

Proof of Theorem 1 Consider the space $A = L^\infty\{(0,T), W^{1,q}(\Omega)\}$ and a closed convex set $B = \{u \in A : 0 \le u \le e^{Mt}\}$, where M is to be determined.

Step 1 For arbitrarily given $u \in B, v \in B$, problem $\Delta\psi = \frac{q}{\varepsilon}(u - v - N(x))$, $\psi|_{\partial\Omega} = \bar{\psi}$ has a unique solution ψ and $|\psi|_{L^\infty((0,T),W^{2,q}(\Omega))} \le C$.

Step 2 For given (u, v) and ψ determined in Step 1, problem

$$k\frac{\partial\theta}{\partial t} - \Delta\theta = H(u, v, \psi, \nabla u, \nabla v, \nabla\psi), \theta|_{t=0} = \theta_0(x), \theta|_{\partial\Omega} = \bar{\theta}$$

has a unique solution θ and $\theta \in L^\infty((0,T), W^{2,q/l_0}(\Omega))$.

Step 3 After we have determined (θ, ψ), let (n, p) be the solutions of following systems:

$$\begin{cases} \frac{\partial n}{\partial t} - D_n\Delta n + \mu_n\nabla n\nabla\psi + \frac{q}{\varepsilon}\mu_n n(n - p - N) + R(n, p, \theta) = 0 \\ \frac{\partial p}{\partial t} - D_p\Delta p - \mu_p\nabla p\nabla\psi - \frac{q}{\varepsilon}\mu_p p(n - p - N) + R(n, p, \theta) = 0 \\ n, p|_{t=0} = n_0(x), p_0(x), n, p|_{\partial\Omega} = \bar{n}(x,t), \bar{p}(x,t) \end{cases} \tag{2.2}$$

We use the super-sub solution method to establish the existence of solutions of (2.2): for the super-sub solution method, we refer to [18] or [19]. Let $(\tilde{n}, \tilde{p}) = (e^{Mt}, e^{Mt})$:

$$\frac{\partial\tilde{n}}{\partial t} - D_n\Delta\tilde{n} + \mu_n\nabla\tilde{n}\nabla\psi - \sup_{0\le p\le e^{Mt}}\Big\{-\frac{q}{\varepsilon}\mu_n\tilde{n}(\tilde{n} - p - N) - R(\tilde{n}, p, \theta)\Big\}$$

$$\ge Me^{Mt} + \frac{q}{\varepsilon}\mu_n e^{Mt}(e^{Mt} - e^{Mt} - N) - l_1r_1 \ge e^{Mt}\Big(M - \frac{q}{\varepsilon}\mu_n\bar{N} - l_1r_1\Big)$$

$$\frac{\partial\tilde{p}}{\partial t} - D_p\Delta\tilde{p} - \mu_p\nabla\tilde{p}\nabla\psi - \sup_{0\le n\le e^{Mt}}\Big\{\frac{q}{\varepsilon}\tilde{p}(n - \tilde{p} - N) - R(n, \tilde{p}, \theta)\Big\}$$

$$\ge Me^{Mt} - \frac{q}{\varepsilon}\mu_p e^{Mt}(e^{Mt} - e^{Mt} - N) - l_1r_1 \ge e^{Mt}(M - l_1l_1)$$

and $(\bar{n}, \bar{p}), (n_0, p_0) \le (1, 1) \le (\tilde{n}, \tilde{p})$, so we have: when $M \ge l_1r_1 + \frac{q}{\varepsilon}\mu_n\bar{N}$, $(\tilde{n}, \tilde{p})$ is the super-solution of (2.2).

Consider $(\underline{n}, \underline{p}) = (0, 0)$:

$$\frac{\partial\underline{n}}{\partial t} - D_n\Delta\underline{n} + \mu_n\nabla\underline{n}\nabla\psi - \inf_{0\le p\le e^{Mt}}\Big\{-\frac{q}{\varepsilon}\underline{n}\mu_n(\underline{n} - p - N) - R(\underline{n}, p, \theta)\Big\}$$

$$= -\inf_{0\le p\le e^{Mt}}\{r(0, p, \theta)l(\theta)\} \le 0$$

The last inequality follows from $r \ge 0$, $l \ge 0$. Similarly we have

$$\frac{\partial\underline{p}}{\partial t} - D_p\Delta\underline{p} - \mu_p\nabla\underline{p}\nabla\psi - \inf_{0\le n\le e^{Mt}}\Big\{\frac{q}{\varepsilon}\mu_p\underline{p}(n - \underline{p} - N) - R(n, p, \theta)\Big\}$$

$$= -\inf_{0\le n\le e^{Mt}}\{r(n, 0, \theta)l(\theta)\} \le 0$$

and $(\underline{n},\underline{p}) = (0,0) \le (\overline{n},\overline{p}), (n_0, p_0)$, so $(\underline{n},\underline{p})$ is the sub-solution of (2.2). From the super-sub solution theorem (see [18] or [19]) we know that (2.2) has a unique solution $(n,p) : (0,0) \le (n,p) \le e^{Mt}, e^{Mt})$.

So we can define an operator $T_1 : T_1(u,v) = (n,p)$. It is easy to prove that $T_1(B) \subseteq B$, in fact $T_1(B)$ is a bounded subset of B. From $\frac{\partial n}{\partial t} - D_n \Delta n + \mu_n n(n-p-N) + R(n,p,\theta) = 0$, $n|_{t=0} = n_0(x)$, $n|_{\partial\Omega} = \bar{n}(x,t)$ and the estimate $0 \le n \le e^{Mt}$, $0 \le p \le e^{Mt}$, $|\psi|_{L^\infty((0,T),W^{2,q}(\Omega))} \le C$, using the L^p estimate for parabolic equation, we obtain $|n|_{L^\infty((0,T),W^{2,q}(\Omega))} \le C$. Using Sobolev compact embedding theorem we claim T_1 is a compact operator. Now we settle to show the continuity of T_1:

Assume $T_1(u_i, v_i) = (n_i, p_i)$, $i = 1,2$, $u = u_1 - u_2$, $v = v_1 - v_2$, $\psi = \psi_1 - \psi_2$, $n = n_1 - n_2$, $p = p_1 - p_2$, $\theta = \theta_1 - \theta_2$. From the equation $\Delta\psi_i = \frac{q}{\varepsilon}(u_i - v_i - N)$, $\psi_i|_{\partial\Omega} = \bar{\psi}$, $i = 1,2$, we have $\Delta\psi = \frac{q}{\varepsilon}(u - v)$, $\psi|_{\partial\Omega} = 0$. Using L^p estimate we have following estimate:

$$|\psi|^q_{W^{2,q}(Q_T)} \le C(|u|^q_{L^q(Q_T)} + |v|^q_{L^q(Q_T)}) \tag{2.3}$$

From the equation that $n_i (i = 1,2)$ satisfy we have:

$$\begin{cases} \frac{\partial n}{\partial t} - D_n \Delta n + \mu_n (\nabla n \nabla \psi_1 + \nabla n_2 \nabla \psi) + \frac{q}{\varepsilon}(n(n_1 + n_2 + p_1 - N) - n_2 p) \\ \quad + R(n_1, p_1, \theta_1) - R(n_2, p_2, \theta_2) = 0, \quad n|_{\partial\Omega} = 0, \quad n|_{t=0} = 0 \end{cases}$$

then we obtain following estimate by using L^p estimate:

$$|n|^q_{W^{2,1}_q(Q_T)} \le C(|\nabla\psi|^q_{L^q(Q_T)} + |n|^q_{L^q(Q_T)} + |p|^q_{L^q(Q_T)} + |\theta|^q_{L^q(Q_T)}) \tag{2.4}$$

and we have similar estimate for p:

$$|p|^q_{W^{2,1}_q(Q_T)} \le C(|\nabla\psi|^q_{L^q(Q_T)} + |n|^q_{L^q(Q_T)} + |p|^q_{L^q(Q_T)} + |\theta|^q_{L^q(Q_T)}) \tag{2.5}$$

From $k\frac{\partial \theta_i}{\partial t} - \Delta\theta_i = H(n_i, p_i, \psi_i, \nabla n_i, \nabla p_i, \nabla\psi_i)$, $\theta_i|_{\partial\Omega} = \theta_0(x)$, we have $k\frac{\partial\theta}{\partial t} - \Delta\theta = H(u_1, v_1, \psi_1, \nabla u_1, \nabla v_1, \nabla\psi_1) - H(u_2, v_2, \psi_2, \nabla u_2, \nabla v_2, \nabla\psi_2)$, $\theta|_{\partial\Omega} = 0$, and following estimate holds by using L^p estimate:

$$|\theta|^q_{W^{2,1}(Q_T)} \le C(|u|^q_{W^{1,0}_q(Q_T)} + |v|^q_{W^{1,0}_q(Q_T)} + |\psi|^q_{W^{1,0}_q(Q_T)}) \tag{2.6}$$

Concluding from (2.3)–(2.6), we have

$$|n|^q_{W^{2,1}_q(Q_T)} + |p|^q_{W^{2,1}_q(Q_T)} \le C(|u|^q_A + |v|^q_A + |n|^q_{L^q(Q_T)} + |p|^q_{L^q(Q_T)})$$

And also we have

$$|n|^q_{L^q(\Omega)} = \int_\Omega \left| \int_0^t n_t(s,x) ds \right|^q dx \le C|n|^q_{W^{2,1}_q(Q_T)}$$

$$|\nabla n|^q_{L^q(\Omega)} = \int_\Omega \left| \nabla \int_0^t n_t(s,x) ds \right|^q dx \le C|n|^q_{W^{2,1}_q(Q_T)}$$

then we have

$$|n|^q_{W^{1,q}(\Omega)} + |p|^q_{W^{1,q}(\Omega)} \le C(|n|^q_{W^{2,1}_q(Q_t)} + |p|^q_{W^{2,1}_q(Q_t)})$$
$$\le C\Big(|u|^q_A + |v|^q_A + \int_0^t (|n|^q_{W^{1,q}(\Omega)} + |p|^q_{W^{1,q}(\Omega)})dt\Big)$$

By using the Gronwall inequality we have $|n|^q_{W^{1,q}(\Omega)} + |p|^q_{W^{1,q}(\Omega)} \le C(|u|_A + |v|_A)$, therefore the continuity of the operator T_1 is proved.

Now by applying the Schauder fixed point theorem, we can show the existence of a solution (n, p, θ, ψ) to system (1.1)–(1.4), (2.1), and using the regularity theory of parabolic equation, we have $(n, p, \theta, \psi) \in C^{2+\alpha, 1+\alpha/2}(Q_T)$ for any $T > 0$.

For uniqueness part, assume there are two solutions $(n_i, p_i, \theta_i, \psi_i)$, $i = 1, 2$, by using the similar steps of proving the continuity of T_1, we have

$$|n|^q_{W^{1,q}(Q_t)} + |p|^q_{W^{1,q}(Q_t)} \le C \int_0^t (|n|^q_{W^{1,q}(Q_s)} + |p|^q_{W^{1,q}(Q_s)})ds$$

Let $g(t) = \int_0^t (|n|^q_{W^{1,q}(Q_s)} + |p|^q_{W^{1,q}(Q_s)})ds$, then we have $dg(t)/dt \le Cg(t)$, $g(t) \ge 0$, $g(0) = 0$, which means $g(t) \equiv 0$, therefore $n = p = 0$, and then $\psi = \theta = 0$, this completes the proof of Theorem 1.

3. Existence and Uniqueness of the Steady State

We consider the following elliptic system:

$$\begin{cases} \Delta\psi = \dfrac{q}{\varepsilon}(n - p - N(x)) \\ \operatorname{div}\vec{J}_n = R(n, p, \theta) \\ \operatorname{div}\vec{J}_p = R(n, p, \theta) \\ \Delta\theta = -H(n, p, \psi, \nabla n, \nabla p, \nabla\psi) \end{cases} \tag{3.1}$$

and boundary condition:

$$n, p, \theta, \psi|_{\partial\Omega} = n_\infty(x), p_\infty(x), \theta_\infty(x), \psi_\infty(x) \tag{3.2}$$

From now on we always assume that Ω is sufficiently narrow in x_1 direction and $\Omega \subset (0, x_0) \times \tilde{\Omega}$. We choose the function space A as $W^{1,\infty}(\Omega)$ and its subset $B = \{u \in A, 0 \le u \le b - ax_1^2\}$, where a, b are positive constants to be determined later.

Theorem 2 *If x_0 is small enough, and $0 \le n_\infty(x)$, $p_\infty(x) \le 1$, $n_\infty(x)$, $p_\infty(x)$, $\theta_\infty(x)$, $\psi_\infty(x) \in C^{2,\alpha}(\bar{\Omega})$, $0 \le r \le r_1$, $0 \le l \le l_1$, $0 \le N(x) \le \bar{N}$, $N(x) \in C^\alpha(\bar{\Omega})$, where r, l, H are Lipschitz continuous functions, $r_1, l_1, \bar{N}$ are positive constants, then the system (3.1)–(3.5) has a unique solution $(n, p, \psi, \theta) \in \{C^{2,\alpha}(\bar{\Omega})\}^4$.*

Proof of Theorem 2 **Step 1** For arbitrarily given $(u,v)\in B\times B$, we can uniquely determine ψ by using equation $\Delta\psi=\frac{q}{\varepsilon}(u-v-N(x)),\ \psi|_{\partial\Omega}=\psi_\infty(x)$.

Step 2 For given (u,v) and the ψ determined in step 1, we can uniquely find a θ satisfying equation: $\Delta\theta=-H(u,v,\psi,\nabla u,\nabla v,\nabla\psi),\theta|_{\partial\Omega}=\theta_\infty(x)$.

Step 3 Assume that n,p are solutions of following equations:

$$\begin{cases}-D_n\Delta n+\mu_n\nabla n\nabla\psi+\frac{q}{\varepsilon}\mu_n n(n-v-N(x))+R(n,v,\theta)=0,\\ \qquad\qquad\qquad\qquad\qquad\qquad n|_{\partial\Omega}=n_\infty(x) & (3.3)\\ -D_p\Delta p-\mu_p\nabla p\nabla\psi-\frac{q}{\varepsilon}\mu_p p(u-p-N(x))+R(n,p,\theta), \quad p|_{\partial\Omega}=p_\infty(x) & (3.4)\end{cases}$$

The proof of the existence of solution of (3.3): consider function $\tilde n=b-ax_1^2$, from $\Delta\psi=\frac{q}{\varepsilon}(u-v-N(x)),\ \psi|_{\partial\Omega}=\psi_\infty(x)$ we have $|\nabla\psi|_{L^\infty(\Omega)}\le C(|\Delta\psi|_{L^\infty(\Omega)}+|\psi|_{L^\infty(\Omega)})\le C|u-v-N(x)|_{L^\infty(\Omega)}\le C+C(b+\bar N)$, where C is some positive constant, then $\tilde n$ satisfies:

$$\begin{aligned}-D_n\Delta\tilde n+\mu_n\nabla\tilde n\nabla\psi-\sup_{0\le v\le a-bx_1^2}&\Big\{-\frac{q}{\varepsilon}\mu_n\tilde n(\tilde n-v-N)-R(\tilde n,v,\theta)\Big\}\\ &\ge 2aD_n-2ax_1\mu_n|\nabla\psi|_{L^\infty}-\frac{q}{\varepsilon}\mu_n\bar N b-r_1l_1\\ &\ge 2aD_n-2ax_1\mu_n(C+C(b+\bar N))-\frac{q}{\varepsilon}\mu_n\bar N b-r_1l_1\end{aligned}$$

If $\tilde n$ is to be a super-solution, it suffices that the last term in above inequalities is positive, i.e., we need following conditions:

$$a\ge\Big(\frac{q}{\varepsilon}\mu_n\bar N+r_1l_1\Big)/D_n,\quad x_0\le D_n/2\mu_n(C+C(b+\bar N)) \tag{3.5}$$

Note that all constants in above inequalities are in independent of x_0. On the boundary we need to have $\tilde n|_{\partial\Omega}\ge n_\infty(x)$. Note that $n_\infty(x)\le 1$, then we need $b-ax_0^2\ge 1$, and it suffices to let

$$b\ge 1+aD_n^2/2\mu_n(C+C(b+\bar N))^2 \tag{3.6}$$

We can easily choose adequate, a,b such that (3.5), (3.6) hold, then $\tilde n=b-ax_1^2$ is a super-solution.

For $\underline{n}=0$, it satisfies

$$\begin{aligned}-D_n\Delta\underline{n}+\mu_n\nabla\underline{n}\nabla\psi-\inf_{0\le v\le b-ax_1^2}&\Big\{-\frac{q}{\varepsilon}\mu_n\underline{n}(\underline{n}-v-N)-R(\underline{n},v,\theta)\Big\}\\ &\le-\inf_{0\le v\le b-ax_1^2}\{r(0,v,\theta)l(\theta)\}\le 0\end{aligned}$$

On the boundary we have $n_\infty(x)\ge 0\ge\underline{n}$, then by virtue of the super-sub solution method (see [20]), there exists a solution n to (3.3) and $0\le n\le b-ax_1^2$.

The existence of solution to (3.4): $\tilde{p} = b - ax_1^2$ satisfies

$$-D_p\Delta\tilde{p} - \mu_p\nabla\tilde{p}\nabla\psi - \sup_{0\le u\le b-ax_1^2}\Big\{\frac{q}{\varepsilon}\mu_p\tilde{p}(u-\tilde{p}-N) - R(u,\tilde{p},\theta)\Big\}$$
$$\ge 2aD_p - 2ax_0\mu_p|\nabla\psi|_{L^\infty} - r_1l_1 \ge 2aD_p - 2ax_0\mu_p(C + C(b+\bar{N})) - r_1l_1 \ge 0$$

In order that the last inequality holds, it suffices that $a \ge r_1l_1/D_p$, $x_0 \le D_p/2\mu_p(C+C(b+\bar{N}))$. On the boundary, $\tilde{p} \ge b - ax_0^2 \ge 1 \ge p_\infty(x)$, and we can choose a large first, and then choose b large, finally choose x_0 small such that all above inequalities hold. Also note that $\underline{p} = 0$ is the sub-solution of (3.4). Again applying the super-sub solution theorem we know that there exists a solution p to (3.4) and $0 \le p \le b - ax_1^2$.

Now let's define an operator $T_2 : T_2(u,v) = (n,p)$, and it can be seen that $T_2(B) \subset B$, in fact $T_2(B)$ is a bounded set of B: from $\Delta\psi = \frac{q}{\varepsilon}(u-v-N)$, $\psi|_{\partial\Omega} = \psi_\infty(x)$, and $0 \le u,\ v \le b - ax_1^2$, we easily deduce the estimate $|\psi|_{W^{1,\infty}} \le C$. From $-D_n\Delta n + \mu_n\nabla n\nabla\psi = -\frac{q}{\varepsilon}\mu_n n(n-v-N) - R(n,v,\theta)$, $n|_{\partial\Omega} = n_\infty(x)$, applying standard L^p estimate and embedding theorem, we can show that $|n|_{W^{1,\infty}} \le C$, and similarly we have $|p|_A \le C$, therefore $T_2(B)$ is bounded in B. We can use the L^p estimate and Sobolev compact embedding theorem to deduce that T_2 is a compact operator.

Now we only need to prove the continuity of operator T_2: assume $T_2(u_i,v_i) = (n_i,p_i)$, $i = 1,2$, $u = u_1 - u_2$, $v = v_1 - v_2$, $n = n_1 - n_2$, $p = p_1 - p_2$, $\theta = \theta_1 - \theta_2$, $\psi = \psi_1 - \psi_2$. For ψ, $\Delta\psi = \frac{q}{\varepsilon}(u-v)$, $\psi|_{\partial\Omega} = 0$, then we have $|\psi|_{W^{2,q}(\Omega)} \le C(|u|_{L^q} + |v|_{L^q})$.

For n, we have

$$D_n\Delta n = \mu_n\Big(\nabla\psi_1\nabla n + \nabla\psi\nabla n_2 + \frac{q}{\varepsilon}(n(n_1+n_2+v_1-N) - n_2v)\Big)$$
$$+R(n_1,v_1,\theta_1) - R(n_2,v_2,\theta_2)$$

By virtue of L^p estimate, we have $|n|_{W^{2,q}} \le C(|\nabla\psi|_{L^q} + |v|_{L^q} + |\theta|_{L^q})$, and similarly for p we have $|p|_{W^{2,q}} \le C(|\nabla\psi|_{L^q} + |u|_{L^q} + |v|_{L^q})$.

For θ, it satisfies

$$\Delta\theta = H(u_2,v_2,\psi_2,\nabla u_2,\nabla v_2,\nabla\psi_2) - H(u_1,v_1,\psi_1,\nabla u_1,\nabla v_1,\nabla\psi_1),\ \theta|_{\partial\Omega} = 0$$

then we have

$$|\theta|_{W^{2,q}} \le C(|\psi|_{W^{1,q}} + |u|_{W^{1,q}} + |v|_{W^{1,q}})$$

Concluding from above estimates we have

$$|n|_{W^{2,q}} + |p|_{W^{2,q}} \le C(|u|_{W^{1,q}} + |v|_{W^{1,q}}) \le C(|u|_A + |v|_A)$$

Let q be sufficiently large, by virtue of Sobolev embedding theorem we have $|n|_A + |p|_A \le C(|u|_A + |v|_A)$, this means the operator T_2 is continuous. According to Schauder's fixed point theorem, we know that T_2 has a fixed point, i.e, (3.1)–(3.2) has a solution (n,p,ψ,θ), furthermore from the regularity theory it follows that $n,p,\psi,\theta \in C^{2,\alpha}(\bar{\Omega})$.

Now let's show the uniqueness part: assume there are two solutions $(n_i, p_i, \theta_i, \psi_i)$, $i = 1, 2$, $n = n_1 - n_2$, $p = p_1 - p_2$, $\theta = \theta_1 - \theta_2$, $\psi = \psi_1 - \psi_2$.

For n, it satisfies $D_n \Delta n - \mu_n(\nabla n \nabla \psi_1 - n_2 \nabla \psi) + R(n_1, p_1, \theta_1) - R(n_2, p_2, \theta_2) = 0$, Multiplying above equation by n and integrating it on Ω, we have $\int_\Omega |\nabla n|^2 \le C \int_\Omega (n^2 + p^2 + \theta_2^2 + |\nabla \psi|^2)$, and similarly for p we have $\int_\Omega |\nabla p|^2 \le C \int_\Omega (n^2 + p^2 + \theta^2 + |\nabla \psi|^2)$. ψ satisfies $\int_\Omega |\nabla \psi|^2 \le C \int_\Omega (n^2 + p^2)$.

For θ, it satisfies $\Delta \theta = H(n_2, p_2, \theta_2, \nabla n_2, \nabla p_2, \nabla \psi_2) - H(n_1, p_1, \theta_1, \nabla n_1, \nabla p_1, \nabla \psi_1)$. Then we have $\int_\Omega |\nabla \theta|^2 \le C \int_\Omega (n^2 + p^2 + \theta^2 + |\nabla \psi|^2 + \delta |\nabla n|^2 + \delta |\nabla p|^2)$. Let δ be sufficiently small and concluding from above estimates we have

$$\int_\Omega (|\nabla n|^2 + |\nabla p|^2 + |\nabla \theta|^2) \le C \int_\Omega (n^2 + p^2 + \theta^2)$$

As we have following inequality

$$\int_\Omega n^2 = \int_\Omega \Big(\int_0^{x_1} n_s ds \Big)^2 dx \le \int_\Omega \Big(\int_0^{x_1} |\nabla n|^2 x_0 \Big) dx \le x_0 \int_\Omega |\nabla n|^2$$

Similar inequalities hold for p, θ, so we have

$$(1 - Cx_0) \int_\Omega (|\nabla n|^2 + |\nabla \theta|^2 + |\nabla p|^2) \le 0$$

Let x_0 be so small that $Cx_0 \le \frac{1}{2}$, then we have $\nabla n = \nabla p = \nabla \theta = 0$. As $n, p, \theta|_{\partial\Omega} = 0$, then we easily deduce that $n = p = \theta = 0$, therefore $\psi = 0$.

This completes the proof of Theorem 2.

4. The Asymptotic Behavior of the Solution of (1.1)–(1.4)+(2.1)

We don't need the condition that x_0 is sufficiently small while we prove the global existence and uniqueness of (1.1)–(1.4)+(2.1). However, the norms of n, p, θ, ψ depend on time T. In order to obtain better estimates, we only need to use $b - ax_1^2$ to replace e^{Mt}, i.e, we have following result:

Theorem 3 *Under the same conditions as in Theorem* 1, *if we further assume* x_0 *is sufficiently small, then* $(1.1) - (1.4) + (2.1)$ *has a unique solution and* $|n|_{L^\infty}$, $|p|_{L^\infty}$, $|\theta|_{L^\infty}$, $|\psi|_{L^\infty} \le C$, *where* C *is independent of* T.

Write the solution obtained in Theorem 2 as $(n^*, p^*, \theta^*, \psi^*)$, the solution obtained in Theorem 1 as (n, p, θ, ψ).

Theorem 4 *If* x_0 *is sufficiently small,* $\max(|n^* - n_0(x)|, |p^* - p_0(x)|, |\theta^* - \theta_0(x)|) \le 1$, $|\bar{n}(x,t) - n_\infty(x)|$, $|\bar{p}(x,t) - p_\infty(x)|$, $|\bar{\theta}(x,t) - \theta_\infty(x)|$, $|\bar{\psi}(x,t) - \psi_\infty(x)| \le Ce^{-\delta_0 t}$, *where* C, δ_0 *are some positive constants, then there exists* $\delta > 0$, $C > 0$ *such that:* $\max(|n - n^*|, |p - p^*|, |\psi - \psi^*|, |\theta - \theta^*|) \le Ce^{-\delta t}$.

Let $\hat{n}, \hat{p}, \hat{\theta}$ be the solutions of the following problem

$$\begin{cases} \frac{\partial \hat{n}}{\partial t} = D_n \Delta \hat{n} - \mu_n \nabla \hat{n} \nabla \psi^* - \frac{q}{\varepsilon} \mu_n n^* (n^* - p^* - N) - R(n^*, p^*, \theta^*) \\ \frac{\partial \hat{p}}{\partial t} = D_p \Delta \hat{p} + \mu_p \nabla \hat{p} \nabla \psi^* + \frac{q}{\varepsilon} \mu_p p^* (n^* - p^* - N) - R(n^*, p^*, \theta^*) \\ k \frac{\partial \hat{\theta}}{\partial t} = \triangle \hat{\theta} + H(n^*, p^*, \psi^*, \nabla n^*, \nabla p^*, \nabla \psi^*) \\ \hat{n}, \hat{p}, \hat{\theta}|_{t=0} = n_0(x), p_0(x), \theta_0(x), \hat{n}, \hat{p}, \hat{\theta}|_{\partial\Omega} = \bar{n}(x,t), \bar{p}(x,t), \bar{\theta}(x,t) \end{cases} \tag{4.1}$$

Lemma 5 *$\exists \delta > 0$ such that: $|\hat{n} - n^*|, |\hat{p} - p^*|, |\hat{\theta} - \theta^*| \le Ce^{-\delta t}$.*

Proof Let $g = b - ax_1^2$, consider super-sub solutions $(\tilde{n}, \tilde{p}, \tilde{\theta}) = (n^* + ge^{-\delta t}, p^* + ge^{-\delta t}, \theta^* + ge^{-\delta t})$ and $(\underline{n}, \underline{p}, \underline{\theta}) = (n^* - ge^{-\delta t}, p^* - ge^{-\delta t}, \theta^* - ge^{-\delta t})$.

Let we check $\tilde{n}$ first

$$\begin{aligned} &\frac{\partial \tilde{n}}{\partial t} - D_n \Delta \tilde{n} + \mu_n \nabla \tilde{n} \nabla \psi^* + \frac{q}{\varepsilon} \mu_n n^* (n^* - p^* - N) + R(n^*, p^*, \theta^*) \\ &\quad = \frac{\partial (ge^{-\delta t})}{\partial t} - D_n \Delta (ge^{-\delta t}) + \mu_n \nabla (ge^{-\delta t}) \nabla \psi^* \\ &\quad \ge e^{-\delta t}(-\delta g) + 2ae^{-\delta t} D_n - 2ax_0 \mu_n e^{-\delta t} |\nabla \psi^*|_{L^\infty} \\ &\quad \ge e^{-\delta t}(2aD_n - \delta g - 2a\mu_n x_0 (C|n^* - p^* - N|_{L^\infty} + C)) \\ &\quad \ge e^{-\delta t}(2aD_n - \delta b - 2a\mu_n x_0 (C(2b + \bar{N}) + C)) \end{aligned}$$

So we only need to assume $\delta \le \min(aD_n/b, \delta_0)$ and $x_0 \le D_n / 2\mu_n (C(2b + \bar{N}) + C)$.

Similarly we can proceed as above to choose adequate δ, x_0 such that $\tilde{p}$ is a super solutions.

For θ, it satisfies

$$k\frac{\partial \tilde{\theta}}{\partial t} - \Delta \tilde{\theta} - H(n^*, p^*, \psi^*, \nabla n^*, \nabla p^*, \nabla \psi^*) = k \frac{\partial (ge^{-\delta t})}{\partial t} - e^{-\delta t} \Delta g \ge e^{-\delta t}(2a - kb\delta)$$

So we only need to choose $\delta \le a/kb$ st $\tilde{\theta}$ is super-solution. And similarly we can choose δ, x_0 such that $\underline{n}, \underline{p}, \underline{\theta}$ are sub-solutions.

For initial values, we have

$$\tilde{n}|_{t=0} = n^* + g(x_1) \ge n^* + 1 \ge n_0(x) = \hat{n}|_{t=0} \ge n^* - g(x_1) = \bar{n}|_{t=0}$$

For boundary values, we have

$$\tilde{n}|_{\partial\Omega} = n_\infty(x) + g(x_1)e^{-\delta t}|_{\partial\Omega} \ge \bar{n}(x,t) \ge n_\infty(x) - ge^{-\delta t}|_{\partial\Omega} = \bar{n}|_{\partial\Omega}$$

And similarly we can check the inequalities of p, θ. Then by virtue of the super-sub solution method theorem we can complete the proof of Lemma 5: Let's write

$\Omega_t=\Omega\times(t,t+1)$, $\Omega^t=\{(x,t):x\in\Omega\}$. As $\hat{n}-n^*$ satisfies equation $\frac{\partial(\hat{n}-n^*)}{\partial t}=D_n\Delta(\hat{n}-n^*)-\mu_n\nabla(\hat{n}-n^*)\nabla\psi^*$, and we already know that $|\hat{n}-n^*|_{\partial\Omega}\le Ce^{-\delta_0 t}$, $|\hat{n}-n^*|_{\Omega^t}\le Ce^{-\delta t}$, then we have $|\hat{n}-n^*|_{W^{2,q}(\Omega_t)}\le Ce^{-\delta t}$. Let q be sufficiently large and by virtue of embedding theorem we have $|\hat{n}-n^*|_{W^{1,\infty}(\Omega_t)}\le Ce^{-\delta t}$. We have same estimates for p and θ, this completes the proof of Lemma 5.

Proof of Theorem 4 Let $W=n(x,t)-\hat{n}(x,t)$, $Z=p(x,t)-\hat{p}(x,t)$, $S=\theta(x,t)-\hat{\theta}(x,t)$, then from (1.2), (4.1) we have

$$\frac{\partial W}{\partial t}-D_n\Delta W+\mu_n(\nabla n\nabla(\psi-\psi^*)+\nabla W\nabla\psi^*)-\frac{q}{\varepsilon}\mu_n[(n+n^*)(n-n^*)+n(p-p^*)$$
$$+N(n^*-n)]+R(n,p,\theta)-R(n^*,p^*,\theta^*)=0,\quad W|_{\partial\Omega}=0$$

therefore

$$|W|_{W_q^{2,1}(\Omega_t)}\le C(|n-n^*|_{L^q(\Omega_t)}+|p-p^*|_{L^q(\Omega_t)}+|\theta-\theta^*|_{L^q(\Omega_t)}+|W|_{L^q(\Omega^t)})$$
$$\le C(|W|_{L^q(\Omega_t)}+|Z|_{L^q(\Omega_t)}+|S|_{L^q(\Omega_t)}+|W|_{L^q(\Omega^t)}+e^{-\delta t})$$

And similar estimate holds for Z; For S, we have

$$k\frac{\partial S}{\partial t}=\Delta S+H(n,p,\theta,\nabla n,\nabla p,\nabla\psi)-H(n^*,p^*,\theta^*,\nabla n^*,\nabla p^*,\nabla\psi^*),\quad S|_{t=0,\partial\Omega}=0$$

Therefore we have

$$|S|_{W_q^{2,1}(\Omega_t)}\le C(|S|_{L^q(\Omega^t)}+|\psi-\psi^*|_{W^{1,q}(\Omega_t)}+|n-n^*|_{W^{1,q}(\Omega_t)}+|p-p^*|_{W^{1,q}(\Omega_t)})$$
$$\le C(|S|_{L^q(\Omega^t)}+|\psi-\psi^*|_{W_q^{2,1}(\Omega_t)}+|\hat{n}-n^*|_{W_q^{2,1}(\Omega_t)}+|\hat{p}-p^*|_{W_q^{2,1}(\Omega_t)}$$
$$+|W|_{W^{1,q}(\Omega_t)}+|Z|_{W^{1,q}(\Omega_t)})\le C(|S|_{L^q(\Omega^t)}+|\psi-\psi^*|_{W^{1,q}(\Omega_t)}+e^{-\delta t}$$
$$+\varepsilon|W|_{W_q^{2,1}(\Omega_t)}+\varepsilon|W|_{L^q(\Omega^t)}+\varepsilon|Z|_{W_q^{2,1}(\Omega_t)}+\varepsilon|Z|_{L^q(\Omega^t)})$$

and from $\Delta(\psi-\psi^*)=\frac{q}{\varepsilon}\{(n-n^*)-(p-p^*)\}$, $(\psi-\psi^*)|_{\partial\Omega}=0$, we have estimate $|\psi-\psi^*|_{W^{1,q}(\Omega_t)}\le C(e^{-\delta t}+|n-n^*|_{L^q(\Omega_t)}+|p-p^*|_{L^q(\Omega_t)})$.

Concluding from above inequalities and letting ε be sufficiently small, we have

$$|W|_{W_q^{2,1}(\Omega_t)}+|Z|_{W_q^{2,1}(\Omega_t)}+|S|_{W_q^{2,1}(\Omega_t)}\le C(|n-n^*|_{L^q(\Omega_t)}+|p-p^*|_{L^q(\Omega_t)}$$
$$+|W|_{L^q(\Omega_t)}+|Z|_{L^q(\Omega_t)}+|W|_{L^q(\Omega^t)}+|Z|_{L^q(\Omega^t)}+|S|_{L^q(\Omega^t)}+e^{-\delta t})$$
$$\le C(|W|_{L^q(\Omega_t)}+|Z|_{L^q(\Omega_t)}+|W|_{L^q(\Omega^t)}+|Z|_{L^q(\Omega^t)}+|S|_{L^q(\Omega^t)}+e^{-\delta t})$$
$$\le Cx_1^{1-1/q}(|\nabla W|_{L^q(\Omega_t)}+|\nabla Z|_{L^q(\Omega_t)})+C(|W|_{L^q(\Omega^t)}+|Z|_{L^q(\Omega^t)}$$
$$+|S|_{L^q(\Omega^t)}+e^{-\delta t})$$

Let x_0 be so small that $Cx_0 \le \frac{1}{2}$, then we have

$$|W|_{W_q^{2,1}(\Omega_t)} + |Z|_{W_q^{2,1}(\Omega_t)} + |S|_{W_q^{2,1}(\Omega_t)} \le C(|W|_{L^q(\Omega^t)} + |Z|_{L^q(\Omega^t)} + |S|_{L^q(\Omega^t)} + e^{-\delta t})$$
$$\le Cx_1^{1/q}(|W|_{L^\infty(\Omega^t)} + |Z|_{L^\infty(\Omega^t)} + |S|_{L^\infty(\Omega^t)} + e^{-\delta t})$$

Let q be sufficiently large, by virtue of Sobolev embedding theorem we have

$$|W|_{L^\infty(\Omega^t)} + |Z|_{L^\infty(\Omega^t)} + |S|_{L^\infty(\Omega^t)}$$
$$\le Cx_1^{1/q}(|W|_{L^\infty(\Omega^t)} + |Z|_{L^\infty(\Omega^t)} + |S|_{L^\infty(\Omega^t)}) + Ce^{-\delta t}$$

Again letting x_0 be small, we have

$$|W|_{L^\infty(\Omega^t)} + |Z|_{L^\infty(\Omega^t)} + |S|_{L^\infty(\Omega^t)} \le Ce^{-\delta t}$$

and then we have

$$|n - n^*|_{L^\infty(\Omega^t)} \le |\hat{n} - n^*|_{L^\infty(\Omega^t)} + |W|_{L^\infty(\Omega^t)} \le Ce^{-\delta t}$$

For similar reason we have

$$|p - p^*|_{L^\infty(\Omega^t)} \le Ce^{-\delta t}, \ |\theta - \theta^*|_{L^\infty(\Omega^t)} \le Ce^{-\delta t}, \ |\psi - \psi^*|_{L^\infty(\Omega^t)} \le Ce^{-\delta t}$$

This completes the proof of Theorem 4.

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References

[1] Van Roosbroeck, Theory of flow of electrons and holes in Germanium and other semiconductors, *Bell System Tech J.* **29** (1950), 560–607.

[2] Selberherr S., Analysis and Simulation of Semiconductor Devices, Springer-Verlag/Wien, 1984.

[3] Wan Yuanming, The analysis and mathematical model of activities of mobiles in semiconductor devices, *Applied Mathematics-A Journal of Chinese Universities* **2** (1987), 228–240 (In Chinese).

[4] Mock M.S., Asymptotic behavior of solutions of transport equations for semiconductor devices, *J. Math. Anal. Appl.*, **49** (1975), 215–255.

[5] Bank R.E., Jerome J. W., Rose D. J., Analytical and Numerical Aspects of Semiconductors Device Modelling Report, 88–11274–2, Bell laboratories (1982).

[6] Gajewski H., On the existence of steady-state carrier distributions in semiconductors, "Probleme und Methoden der Mathematicschen Physik", *Teubner-Texte Zur Mathematik,* **63** (1983), 76–82.

[7] Seidmann T. I., Steady-state solutions of diffusion-reaction systems with electrostatic convection, *Nonlinear Analysis,* **4** (1981), 623–637.

[8] Gajewski H., On uniqueness and stability of steady state carrier distributions in semiconductors, EQUADIFF 6, Lecture Notes in Mathematics 1192, Springer-Verlag (1986).

[9] Mock M.S., On equations describing steady-state carrier distributions in a semiconductor device, *Comm. Pure. and Appl. Math.,* **25** (1972), 781–792.

[10] Mock M.S., An example of nonuniqueness of stationary solutions in semiconductor device Models, COMPL **3** (1982), 165–174.

[11] Mock M.S., Analysis of Mathematical Models of Semiconductor Devices, Dublin: Boole Press (1983).

[12] Gajewski H., On existence, uniqueness and 0 behavior of solutions of the basic equations for carrier transportation in semiconductors, *Z. Angew. Math. Mech,* **65** (1985), 101–108.

[13] Mock M.S., An initial value problem from semiconductor theory, *SIAM J. Math. Anal,* **5** (1974), 597–612.

[14] Gajewski H. and Groger K., On the basic equations for carrier transport in semiconductors, *J. of Math. Anal. Appl.,* **113** (1986), 12–35.

[15] Seidmann T. I., Time dependent solutions of a nonlinear system arising in semiconductor theory–II. boundaries and periodicity, *Nonlinear Analysis,* **10** (1986), 491–502.

[16] Seidmann T. I., Trorianiello G. I., Time dependent solutions of a nonlinear system arising in semiconductor theory, *Nonlinear Analysis,* **9** (1985), 1137–1157.

[17] Liang J., On the semiconductor's system, *Journal of Partial Differential Equations,* **5** (1992), 69–78.

[18] Liang J., The reaction-diffusion system without quasi-monotone conditions, *Acta Scientiarum Naturalium Universitatis Pekinensis,* **3** (1987), 9–20.

[19] Ye Qixiao and Li Zhengyuan, Introduction to Reaction-Diffusion Equation, Science Press, Beijing, 1990 (in Chinese).

[20] Smoller J., Shock Waves and Reaction-Diffusion Equations, Springer-Verlag, 1983.