

Study on Model of Heat and Moisture Transfer of Fractal Polyesters

Jiquan Fu*, Tianwen Chen

Beijing Institute of Fashion Technology, Beijing 100029, China

Abstract

The dynamic moisture absorption of fractal polyester was studied at a constant temperature (30°C) and humidity (80% RH). The model of heat and moisture transfer was then built based on the heat and mass transfer theories. Finite element method was used to solve the model. By comparing the moisture absorption predicted by the mechanism model and the experimental data, we concluded that variations trend of both were consistent. Besides, fields of temperature and humidity with different thickness of fractal polyester were predicted and analyzed by this model.

Keywords: Fractal Polyester; Moisture Transfer; Model; Humidity Field

1 Introduction

In recent years, researchers make great effort to develop fractal fibers in order to essentially make it surpass the nature fibers. Researchers from America and Japan have been studying the fractal structure of fibers, which is related to the correspondence between similarities of convex-concave structure on fiber surface and the shade in the nature [1]. The fractal polyester developed by Professor Xushan Gao imitates some characteristics of nature fibers. Its natural twist is a kind of fractal structure besides the oriented crystallization indicated by winding tension in the process of fractal polyester forming, with some induced crystallization caused by other conditions which was added in the process. The condition of necking position was controlled to form a 7×21 nm nano crystal structure. It's a nanoparticle gene, which can cause twists. According to fractal theory, it can scale-up 104 times on a self-similarity basis, which creates fractal twists from inside to outside (which extremely similar to twists of nature fibers). As a result, it can easily shape a “dragon wrapped around column” structure in blending fiber, which makes the synthetic fiber to essentially achieve nature structure.

Heat and moisture transfer performance of the “dragon wrapped around column” textile (fractal polyester for short) made of fractal polyester and differential fiber is one of the focal points in this study. Mass production of the fractal polyester provides a guarantee for the study of material performance. Stressed on the heat and moisture transfer model of fractal polyester, this paper

*Corresponding author.

Email address: fujq010@sina.com (Jiquan Fu).

aims to develop a model which can reveal transfer performance objectively, which has a great value to adjust the fabric structure further and develop new types of fractal polyester fabrics.

2 Experiments

2.1 Materials

Plain weave fractal polyester fabric (30% fractal polyester and 70% viscose) was provided by Beijing top new group. Its main parameters are shown in Table 1. Fabrics were purified by acetic ester solution to remove impurities such as oily wax, and then it was washed by soap bath and warm water, dried and gone through setting before it was used for this study.

Table 1: Main Fabric Parameters

Name	Structure	Thickness (mm)	Count (tex)	Area Density (g/m ²)
Polyester Fabric	Knitting	1	50D/40	196

2.2 Instruments and Equipments

A Temperature Humidity Test Chamber, (type: SDH05011) was used to do the experiments, which has the following technical features:

- (1) Precision of temperature measurement: $\pm 0.3\%$ F.S.
- (2) Precision of humidity measurement: $\pm 0.5\%$ F.S (Room Temperature between 20°C and 90°C).

2.3 Methods

Experimental Methods: According to GB/T9995-1997 “Determination of moisture content and moisture regain of textile-Oven-drying method”, low temperature humidity test chamber method was adopted to measure moisture absorption of fibers. Based on low temperature humidity test chamber, self-made moisture testing system was employed to measure dynamic moisture absorption of fabric. Conditions: temperature 35°C, wind speed 0.4 m/s, humidity RH80%.

Calculation method of fabric moisture content: Weigh the dry weight of fabric after baking in the oven for 2.5 hrs, label as W_0 , then put the fabric in the given condition, and record the value W on the electronic scale every 10 minutes. The formula of fabric moisture content is:

$$\text{Moisture content} = \frac{W - W_0}{W} \times 100\%$$

3 Model of Heat and Moisture Transfer for Fractal Polyesters

According to mass and energy conservation, coupling model of heat and moisture transfer between human body, textile and environment is built to describe the transfer case of fractal polyester fabrics. The first model was proposed by Henry [2], then developed by Nordon [3], Li [4] and Luo [5], etc. Now the model can well describe heat and moisture transfer mechanism of fabrics. We utilize the modified heat and moisture coupling model to discuss the moisture transfer problem of fractal polyester.

3.1 Model of Heat Transfer

Heat transfer of microclimate not only includes heat conduction, convection and radiation, but also accompanied by evaporation-condensation. Literature [6] uses limited space convection calculation to deal with effects of convection on microclimate area, and converts it into equivalent thermal conductivity, whilst considering the evaporation-condensation phenomenon as inner heat source style.

Regarding the garment within microclimate as a thin plate, where A is end-face area, L thickness, k equivalent thermal conductivity, c heat capacity, ρ density, q inner heat source. As is shown in Fig. 1, selecting dx as volume element, and according to conservation of heat energy, net increment of heat flowing into and out of volume element during dt and heat productivity of inner heat source equals to the internal energy increment of volume element.

$$\begin{matrix}
 \left(\begin{array}{l} \text{net increment of} \\ \text{heat flowing into} \\ \text{and out of volume} \\ \text{element} \end{array} \right) & + & \left(\begin{array}{l} \text{heat product} \\ \text{ivity of} \\ \text{inner heat} \\ \text{source} \end{array} \right) & = & \left(\begin{array}{l} \text{increm ent of} \\ \text{internal energy} \\ \text{during the} \\ \text{micro unit} \end{array} \right) & (1) \\
 \text{I} & & \text{II} & & \text{III}
 \end{matrix}$$

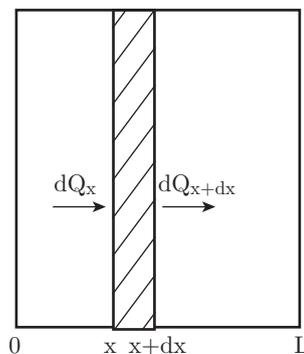


Fig. 1: Fluxional volume element of fabric heat transfer

(1) Heat flowing into the volume element

Heat flowing into the unit along the x section of fabric thickness in time dt can be written as

$$dQ_x = q_x A dt \tag{2}$$

Heat flowing out of $x dx$ can be written as

$$dQ_{x+dx} = q_{x+dx} A dt \quad (3)$$

Where q_x is the thermal flux of volume element area along x direction when fabric thickness is x , in which we obviously have:

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx \quad (4)$$

$$q_x = -k \frac{\partial T}{\partial x} \quad (5)$$

According to Fourier's law, we know that net increment of heat flowing into and out of volume element during dt is

$$\begin{aligned} I &= dQ_x - dQ_{x+dx} = q_x A dt - q_{x+dx} A dt \\ &= q_x A dt - \left(q_x + \frac{\partial q_x}{\partial x} dx \right) A dt = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) A dx dt \end{aligned} \quad (6)$$

(2) Heat productivity of inner heat source in volume element is

$$II = q A dx dt \quad (7)$$

(3) Internal energy increment of volume element during time dt

$$III = c_v \rho \frac{\partial T}{\partial t} A dx dt \quad (8)$$

substitute (6), (7), (8) into (1), we can have

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) A dx dt + q A dx dt = c_v \rho \frac{\partial T}{\partial t} A dx dt \quad (9)$$

After arrangement we can get:

$$k \frac{\partial^2 T}{\partial x^2} + q = c_v \rho \frac{\partial T}{\partial t} \quad (10)$$

3.2 Model of Moisture Transfer

Similar to heat transfer, moisture transfer in microclimate is a mass transfer process to be derived by the difference of relative humidity, which may correspond with phase change phenomena. When humid air comes into saturation, evaporation-condensation phenomenon will appear and will be considered as an inner mass source, because the partial pressure of water vapor in humid air is limited by saturated vapor pressure.

Regarding the garment within microclimate as a thin plate, where A is end-face area, L thickness, D_a equivalent diffusion coefficient, m_i inner mass source. As is shown in Fig. 2, selecting dx as volume element, according to Mass Conservation law, net increment of mass flowing into

and out of volume element during dt and mass productivity of inner mass source equals to mass increment of volume element.

$$\begin{matrix}
 \left(\begin{array}{c} \text{net increment} \\ \text{of mass flowing} \\ \text{into and out} \\ \text{of volume} \end{array} \right) & + & \left(\begin{array}{c} \text{mass produ} \\ \text{ctivity of} \\ \text{inner mass} \\ \text{source} \end{array} \right) & = & \left(\begin{array}{c} \text{mass incre} \\ \text{ment of} \\ \text{micro} \\ \text{unit} \end{array} \right) \\
 \text{I} & & \text{II} & & \text{III}
 \end{matrix} \tag{11}$$

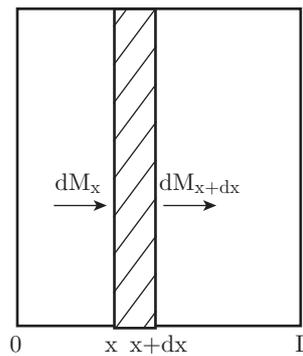


Fig. 2: Fluxional volume element of fabric mass transfer

Similar to deduction of mass(moisture) transfer, we have:

(1) According to Mass Conservation law, net increment of mass flowing into and out of volume element during dt is

$$I = \frac{\partial}{\partial x} \left(D_a \frac{\partial C}{\partial x} \right) A dx dt \tag{12}$$

(2) Mass productivity of inner mass can be written as

$$II = m_i A dx dt \tag{13}$$

(3) Mass increment of volume element during dt

$$III = \frac{\partial C}{\partial t} A dx dt \tag{14}$$

After arrangement we can get:

$$D_a \frac{\partial^2 C}{\partial x^2} + m_i = \frac{\partial C}{\partial t} \tag{15}$$

3.3 Solution of the Model

Both heat transfer and mass transfer exist in transfer problems of the same fabric. As their models are coupled with each other, we can use methods such as finite element to obtain the numerical solution.

3.3.1 Initial Conditions and Boundary Conditions

Put the fabric into constant temperature and humidity (T0, C0) at initial time. Given the diffusion of moisture and heat is fast enough, each kind of temperature and humidity of the fabric surface is equal to that in the air when it comes to heat and moisture balance, and the distribution of temperature and humidity in the fabric is symmetric about xL/2, so we can conclude the following conditions.

Initial conditions:

$$T|_{t=0} = T(0, x) = T_0, \quad (0 < x < L)$$

$$C|_{t=0} = C(0, x) = C_0, \quad (0 < x < L)$$

Boundary conditions:

$$T|_{x=0} = T(t, 0) = T_0, \quad \left. \frac{\partial T}{\partial x} \right|_{x=L/2} = 0$$

$$C|_{x=0} = C(t, 0) = C_0, \quad \left. \frac{\partial C}{\partial x} \right|_{x=L/2} = 0$$

3.3.2 Calculation of Using Numerical Analysis

Heat and mass transfer model is a non-steady state process; application of numerical analysis method is quite complex, while numerical method based on finite element and finite difference is very effective [7]. We adopted the finite difference method, dividing the fabric layer into a limited number of grid cells, and then obtain temperature and humidity values of each grid cell nodes in the fabric layer by numerical calculation method, as shown in Fig. 3.

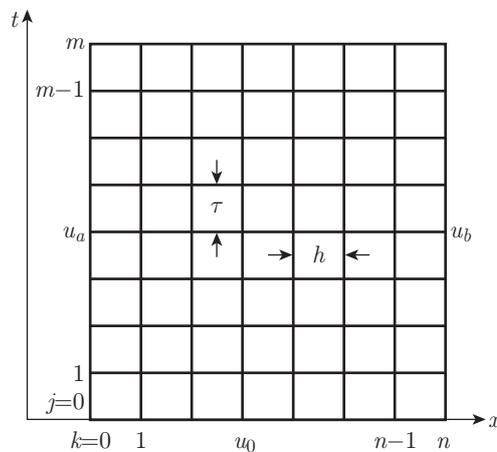


Fig. 3: Finite difference grid

The heat and moisture transfer model in this article belongs to parabola partial differential equation, which can be solved by many difference methods, such as classical formats (including explicit and implicit), Crank-Nicholson formats, three explicit schemes, and three implicit schemes. Six-point format is used to solve this equation, because it not only has higher precision

but is unconditionally stable as an implicit scheme. Use the average of central divided difference at $((j+1)/2, k)$ approximate substitute first order derivative of t , we can conclude

$$\frac{\partial u}{\partial t} \Big|_{j+1/2} = \frac{u_{j+1,k} - u_{j,k}}{2(\tau/2)} + O(\tau^2) \tag{16}$$

Use the average of central divided difference at j and $j+1$ approximate substitute second derivative of x , we can conclude

$$\frac{\partial^2 u}{\partial x^2} \Big|_{j+1/2} = \frac{1}{2} \left[\frac{u_{j+1,k+1} - 2u_{j+1,k} + u_{j+1,k-1}}{h^2} + \frac{u_{j,k+1} - 2u_{j,k} + u_{j,k-1}}{h^2} \right] + O(h^2) \tag{17}$$

Discard truncation error, consider the boundary conditions, put them into the model equation, and we can obtain the linear set of tridiagonal equations after arrangement. Pursuit method can be adopted and the software (QBasic) can be used to solve the equations.

4 Test of the Model and Analysis of Simulation Results

4.1 Determination of Model Parameters

In order to verify the model, numerical value by computer calculation can be predicted to educe the values of temperature and humidity in the fabric. The model can be tested by comparing coincidence degree between the calculation values and experimental results. Reasonable parameters in the calculation should be determined at first. The basic parameters of the fabric are shown below.

(1) Basic physical parameters

Basic physical parameters of the fabric are shown in Table 2.

Table 2: Basic physical parameters of the fabric

Names	Fractal polyester	Viscose
Fabric heat conduction k (w/m. °C)	0.084	0.065
Fiber heat capacity c_v (kJ/m ³ · K)	$(1531 + 4184w)/(100 + w)$	$(1785 + 4184w)/(100 + w)$
Fiber density ρ (kg/m ³)	1220	1500
Diffusion coefficient of water in the fiber D_f (m ² /s)	3.9×10^{-13}	5.9×10^{-13}

Where w is equilibrium moisture regain of the fiber.

(2) Other physical parameters

Porosity of the fabric $\varepsilon = 0.9$;

Diffusion coefficient of water vapor in the air $D_{a0} = 2.49 \times 10^{-5}$ m²/s;

Heat capacity of the water vapor $c_v = 4184$ kJ/m³ · K.

4.2 Comparison of Model Simulation Values and Experimental Results

Experimental results of fiber water absorption and model simulation values at different times are shown in Fig. 4. By comparing experimental results and simulating the data, we can find that their changing trend seems consistent, which verifies the compatibility of the model of coupled heat and moisture transfer. As many factors can influence the process of heat and moisture transfer in the fabric, we make some simplifications which may bring some errors, and cause the difference between the model and the fact, so there is a certain bias between experimental and predicted data. But it still can satisfy the need of necessary regularity of the model simulation analysis at current situation.

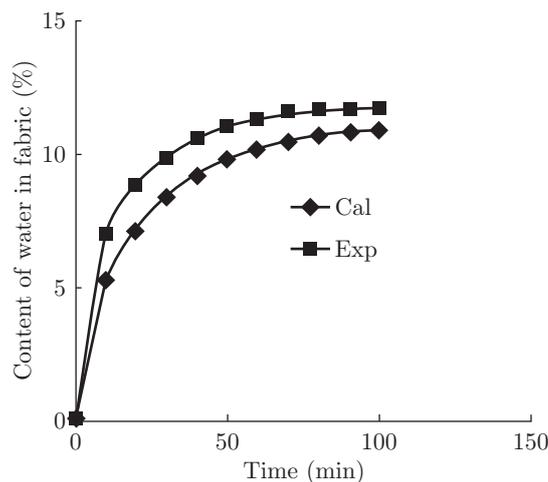


Fig. 4: Comparison of experimental results and predicted values content of water in fabric (%)

4.3 Simulation Results of the Model

Temperature and humidity distribution in the textile can be shown by three-dimensional graphics according to simulation results of the model.

(1) Temperature distribution in the fabric

From Fig. 5, it can be seen that the heat transfer of the fabric reaches equilibrium after 10 mins, in which temperature at each point shows parabolic with time change—temperature is low at the center of the fabric layer while high at the boundary. We only simulate the heat transfer between fabric and environment while convection and fabric moisture differential thermal at the boundary layer are not concluded in this model.

(2) Humidity distribution in the fabric

From Fig. 6, it can be understood that the distribution of moisture concentration (which has been converted into water content) in the fabric changes with time exponentially. The reason is that the moisture concentration was increased rapidly at the beginning of the moisture transfer, at the time water enters into the fabric mainly by diffusion, and the driving potential is the difference of relative humidity between within the fabric and the outside air. When moisture concentration changes are higher, the weak driving potential is obtained so that water transfer

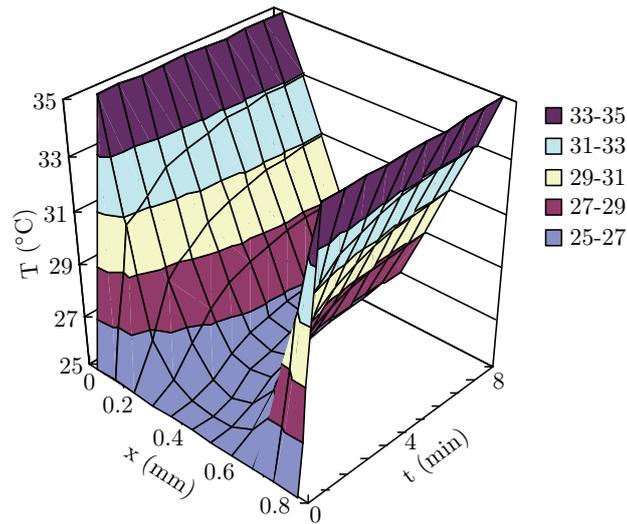


Fig. 5: Temperature distribution in the fabric

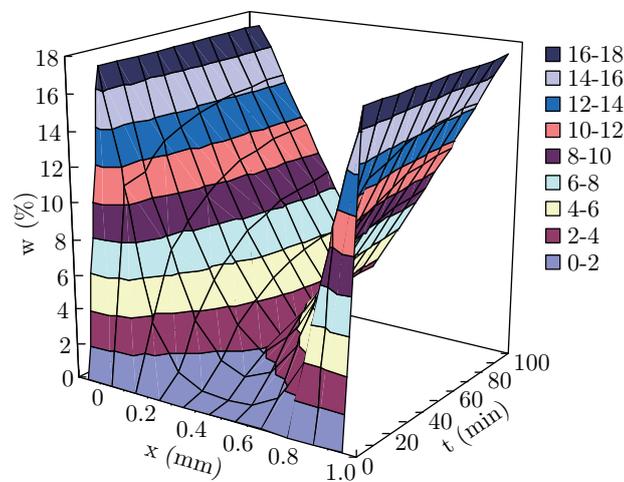


Fig. 6: Humidity distribution in the fabric

rate decreases. Moisture equilibrium of fractal polyester achieves equilibrium after 100 min, but the moisture concentration at the center is lower than that at the outer surface.

5 Conclusion

(1) Based on previous research, the model of heat and moisture transfer is built to be used for simulation of fractal fabric moisture transfer, and the changing trend of the simulation values of fabric moisture absorption agrees well with that of experimental results.

(2) As factors influencing the process of heat and moisture transfer in the fabric are numerous and complex, we made some simplifications in order to solve the model. The results of the model solution show that finite element method is in effect to obtain the numerical solution. The temperature and humidity distribution of the fabric can be obtained by the model and the solution method.

References

- [1] Gao XS, Tong Y, Zhuang Y and Hong JQ. The fractal structure of natural fibers and the development of fiber with fractal structure, *China Synthetic Fiber Industry*, 2000, 23(4): 35-38.
- [2] Henry PSH. The Diffusion of Moisture and Heat Through, *Textiles*. *Discuss. Faraday Soc.*, 1948; 3: 243-257.
- [3] David HG and Nordon P. Case Studies of Coupled Heat and Moisture Diffusion in Wool Beds. *Textile Res. J.*, 1969; 39: 166-172.
- [4] Li Y and Holcombe BV. A Two-stage Sorption Model of the Coupled Diffusion of Moisture and Heat in Wool Fabric. *Textile Res. J.*, 1992; 64: 211-217.
- [5] Luo ZX, et al. Heat and Moisture Transfer with Sorption and Condensation in Porous Clothing Assemblies and Numerical Simulation. *International Journal of Heat and Mass Transfer* , 2000; 43: 2989-3000.
- [6] Franworth B. Mechanisms of Heat Flow through Clothing Insulation. *Textile Res. J.*, 1983; 53: 717-725.
- [7] Li FZ. Numerical Simulation of Coupled Heat and Mass Transfer in Hygroscopic Fabrics Considering the Influence of Atmospheric Pressure. *Numerical Heat Transfer, Part B*. 2004; 45: 249-262.