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Pricing Stocks with Trading Volumes

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Abstract. The present paper proposes a new framework for describing the stock price dynamics. In the traditional geometric Brownian motion model and its variants, volatility plays a vital role. The modern studies of asset pricing expand around volatility, trying to improve the understanding of it and remove the gap between the theory and market data. Unlike this, we propose to replace volatility with trading volume in stock pricing models. This pricing strategy is based on two hypotheses: A price-volume relation with an idea borrowed from fluid flows and a white-noise hypothesis for the price rate of change (ROC) that is verified via statistic testing on actual market data. The new framework can be easily adopted to local volume and stochastic volume models for the option pricing problem, which will point out a new possible direction for this central problem in quantitative finance.

AMS subject classifications: 91G30, 91G20, 62G10

Key words: Asset pricing, volatility, trading volume, stock price model, white noise.

1 Introduction

The equity (stocks) and their derivatives (options, futures, etc.) are the most traded financial products in markets. Developing effective mathematical models for stock and option prices plays a vital role in the financial industry and the quantitative finance research. Using stochastic processes to model stock returns has a long history, and it dates back to Bachelier's pioneer work in 1900 [2], where the Brownian motion was used for describing the uncertainty of the stock price dynamics. In 1965, Samuelson [16] adopted

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Bachelier's idea with a modification to evaluate the warrant, a financial derivative product. Samuelson introduced the geometric Brownian motion (GBM) for the dynamics of a stock price

$$dS = \mu S dt + \sigma S dB, \tag{1.1}$$

where $S = S(t)$ is the stock price, $B = B(t)$ is the standard Brownian motion, and μ and σ are two parameters standing for the drift rate and the volatility respectively. In 1973, the GBM is used by Black-Scholes [4] and Merton [15] in their celebrated model (known as the Black-Scholes or Black-Scholes-Merton model, BS or BSM) for option pricing. The BS model gained tremendous success in the option market and dominated this area for decades, and the GBM and its variants have been the fundamentals for asset pricing ever since. The GBM (1.1) can be rewritten as

$$\frac{dS}{S} = \mu dt + \sigma dB, \tag{1.2}$$

and if letting ΔS denote the change of S in a small time interval Δt, we have the discrete-time version of (1.2)

$$\frac{\Delta S}{S} \sim \mathcal{N}(\mu \Delta t, \sigma^2 \Delta t), \tag{1.3}$$

where $\mathcal{N}$ is the normal distribution, and thus this model is also known as the log-normal model, see [6, 12].

The volatility σ plays a unique role in the BS model, and its nature and calculation methods receive much attention in the studies and financial practice. Roughly speaking, σ measures how large the stock price S fluctuates around its mean value in the statistical sense. Larger volatility indicates the stock price is less predictable and thus is riskier. Though the importance of volatility, there are different understandings and calculation methods. For instance, there are two often-used terminologies, the historical volatility, which is calculated based on the historical stock prices and represents the degree of variability; and the implied volatility, which is calculated by the inverse problem of finding this parameter in the BS equation with knowing the option price.

The above observations suggest that volatility may not be as essential as assumed, though seemingly important and widely accepted in practice. A more in-depth investigation of the stock price dynamics could improve our understanding of its nature and benefit stock trading strategies and the option pricing modeling. The present work is our first attempt in this direction, and we propose to use the trading volume as an alternative for the volatility. There are several reasons for doing so:

(i) The trading volume is a more explicit factor than volatility, its magnitude and meaning are self-explanatory, and no confusion or different values should be involved.

(ii) There are extensive theoretical and empirical studies on the volume-volatility relation, it is shown that the return volatility and the trading volume are positively correlated [1, 3, 13, 18].

(iii) The price and volume are the only two essential quantities that associates to the stock trading, we believe the market is efficient, and all the information, participants' behavior, liquidity and other factors that affects the stock price dynamics should be reflected and encoded in these two factors.

Therefore, we propose a new approach of modeling stock price by taking the trading volume as the central ingredient.

The rest of this paper is organized as follows. In Section 2, we first give two hypotheses of the model, the first one is the price-volume relation by making an analogy between the cash flow of stock transactions and the flow of a fluid, and the second one is that the price rate of change (ROC) is a white-noise process, which we examined by statistic testing on actual market data. Under the two hypotheses, we derived the model for stock price dynamics. Our model includes the classical GBM as a particular case. In Section 3, we discuss the applications of the new model to the option pricing problems. The proposed model is easily adapted to the local and stochastic volume models for options. The empirical studies and statistical testing for the Hypothesis (H2) are given in the Appendix A.

2 Modeling stock price with trading volume

2.1 Trading volume and net cash flow

The market capitalization of a company is the market price multiplied by the number of outstanding shares. There are two different kinds of shares for a listed company, restricted shares that are non-transferable and free-float shares that are tradable in the stock exchange

$$\text{Free float} = \text{Outstanding shares} - \text{Restricted shares}.$$

We shall assume the numbers of outstanding, restricted and free-float shares are not changed in the time period under consideration, namely, there is no share splits or consolidation.

Our approach is to make an analogue of the stock trading with the fluid flow. We may regard the market capitalization as a tank of fluid, and the transaction will drive the change of the stock price and thus the change of the market capitalization. The trading volume of the transactions can be regarded as the volume of the fluid that is exchanged between the tank and its environment.

The market price is driven by the cash flow, and thus we consider the net cash flow of a stock in the time period $[t,t+\Delta t)$. Assume the transactions take place continuously over time, and let $S(t)$ denote the stock price and $v(t)$ denote the average volume, i.e. instantaneous trading volume per unit time. If $\tilde{v}(t)$ denotes the trading volume over $[0,t]$, we then have

$$v(t) = \lim_{\Delta t \to 0} \frac{\tilde{v}(t+\Delta t) - \tilde{v}(t)}{\Delta t}. \tag{2.1}$$

A related and vital quantity is the trading intensity defined as

$$\hat{v}(t) := \lim_{\Delta t \to 0^+} \frac{\tilde{v}(t+\Delta t) - \tilde{v}(t)}{\sqrt{\Delta t}} = \lim_{\Delta t \to 0^+} \frac{1}{\sqrt{\Delta t}} \int_t^{t+\Delta t} v(\tau) d\tau. \tag{2.2}$$

We shall assume the trading volume $\tilde{v}(t)$ is a $C^{1/2}$ function, that is a Hölder continuous function of $1/2$ order. Hence, v is in the Sobolev space $H^{-1/2}$ and $\hat{v}(t)$ is a continuous function. Noting that from Eqs. (2.1)-(2.2), we have

$$\hat{v}(t) = \lim_{\Delta t \to 0^+} \left[\sqrt{\Delta t} v(t) \right], \tag{2.3}$$

which is a bounded function of t according to the above assumption of $v(t)$ and $\tilde{v}(t)$. The cash outflow is the total capital of the seller taking from the stock

$$\text{cash outflow} = \int_t^{t+\Delta t} S(\tau) v(\tau) d\tau.$$

After these transactions, the stock price becomes $S(t+\Delta t)$, and the wealth of the stock buyer is the product of this price and the number of traded shares, which is the cash inflow,

$$\text{cash inflow} = S(t+\Delta t) \int_t^{t+\Delta t} v(\tau) d\tau.$$

Therefore, the net cash flow in the interval $[t, t+\Delta t)$ is given by the amount of cash inflow subtracts the outflow, which is

$$\text{net cash flow} = \int_t^{t+\Delta t} [S(t+\Delta t) - S(\tau)] v(\tau) d\tau. \tag{2.4}$$

2.2 Model hypotheses

The stock price $S(t)$ varies due to the time-value as well as driven by the cash-flow. The time value can be evaluated as an exponential growth with the return rate μ, and the effect of the cash-flow should be calculated per-share. Based on the analysis in the previous section, we shall make the following and our main hypothesis on the stock price dynamics:

(**H1**) The stock price satisfies

$$S(t) = S(t_0) e^{\mu(t-t_0)} + \frac{1}{\boldsymbol{V}_0} \int_{t_0}^t [S(t) - S(\tau)] v(\tau) d\tau, \tag{2.5}$$

where $\boldsymbol{V}_0$ denotes the number of free-float shares.

The second hypothesis concerns the price rate of change (ROC), a commonly-used indicator in stock investment, for which we shall assume

(**H2**) The price rate of change (ROC) follows a white noise process

$$\lim_{\Delta t\to 0^+}\frac{S(t+\Delta t)-S(t)}{S(t)\sqrt{\Delta t}}=\frac{dB(t)}{dt}. \tag{2.6}$$

The study of the stock dynamics is the central theme in asset pricing and has received great attention. Granger and Morgenstern [10] showed that the short-run movements of stock price obey the simple random walk hypothesis. Later on, Fama's seminal work [9] proved the validity of the Efficient Markets Hypothesis, which became the foundation of the asset pricing theory. These suggest that stochastic processes can be used to model the short-run stock changes. In 1963, Mandelbrot [14] pointed out that the stock changes do not follow a Gaussian but a steady process with an unbounded variation. The white noise in our hypothesis (H2) is indeed such a process. We conducted an empirical study of this hypothesis using actual market data and tested the hypothesis through statistical hypothesis testing procedures. Details are provided in Appendix A.

2.3 The model

To establish our model for the stock price, we need the following result.

Lemma 2.1. *Under the hypothesis (H2), we have*

$$\lim_{\Delta t\to 0}\frac{\displaystyle\int_t^{t+\Delta t}\big(S(t+\Delta t)-S(\tau)\big)v(\tau)d\tau}{\boldsymbol{V}_0\Delta t}=\frac{\hat{v}(t)}{\boldsymbol{V}_0}S(t)\frac{dB(t)}{dt} \tag{2.7}$$

in the sense of distribution.

Proof. Let $\mathbf{I}_{[t,t+\Delta t]}(\cdot)$ denote the indicator (or, characteristic) function of the interval $[t,t+\Delta t]$. Assume $f(x)$ is a continuous function, we shall show that

$$\lim_{\Delta t\to 0}\int_0^\infty f(x)\frac{\mathbf{I}_{[t,t+\Delta t]}(x)}{\Delta t}dx=f(t),\quad \forall t>0. \tag{2.8}$$

Note that

$$\begin{aligned}&\left|\int_0^\infty f(x)\frac{\mathbf{I}_{[t,t+\Delta t]}(x)}{\Delta t}dx-f(t)\right|\\=&\left|\int_0^\infty [f(x)-f(t)]\frac{\mathbf{I}_{[t,t+\Delta t]}(x)}{\Delta t}dx\right|\\\le&\int_0^\infty |f(x)-f(t)|\frac{\mathbf{I}_{[t,t+\Delta t]}(x)}{\Delta t}dx.\end{aligned}$$

Since $f(x)$ is continuous at t, for any $\varepsilon>0$ there exists a $\delta>0$ such that $|f(x)-f(t)|<\varepsilon$ for $|x-t|\le\Delta t<\delta$. Therefore,

$$\left|\int_0^\infty f(x)\frac{\mathbf{I}_{[t,t+\Delta t]}(x)}{\Delta t}dx-f(t)\right|<\varepsilon,\quad \forall \Delta t<\delta,$$

and Eq. (2.8) follows. An application of this equation yields

$$\begin{aligned}
&\lim_{\Delta t\to 0}\frac{\displaystyle\int_t^{t+\Delta t}\big(S(t+\Delta t)-S(\tau)\big)v(\tau)d\tau}{\boldsymbol{V}_0\Delta t}\\
=&\lim_{\Delta t\to 0}\frac{S(t+\Delta t)}{\boldsymbol{V}_0\Delta t}\int_{-\infty}^{\infty}v(\tau)\mathbf{I}_{[t,t+\Delta t]}(\tau)d\tau\\
&\quad-\lim_{\Delta t\to 0}\frac{1}{\boldsymbol{V}_0\Delta t}\int_{-\infty}^{\infty}S(\tau)v(\tau)\mathbf{I}_{[t,t+\Delta t]}(\tau)d\tau\\
=&\frac{1}{\boldsymbol{V}_0}\lim_{\Delta t\to 0}\big\{v(t)\big[S(t+\Delta t)-S(t)\big]\big\}.
\end{aligned}$$

Now, by the Hypothesis (H2), we have

$$\begin{aligned}
&\lim_{\Delta t\to 0^+}\big\{v(t)\left[S(t+\Delta t)-S(t)\right]\big\}\\
=&S(t)\lim_{\Delta t\to 0^+}\left\{\left[v(t)\sqrt{\Delta t}\right]\frac{[S(t+\Delta t)-S(t)]}{S(t)\sqrt{\Delta t}}\right\}\\
=&S(t)\lim_{\Delta t\to 0^+}\left[v(t)\sqrt{\Delta t}\right]\lim_{\Delta t\to 0^+}\frac{S(t+\Delta t)-S(t)}{S(t)\sqrt{\Delta t}}\\
=&\hat{v}(t)S(t)\frac{dB(t)}{dt},
\end{aligned}$$

where we used the trading intensity given in (2.3). In the above, we assumed $\Delta t>0$ and taking $\Delta t\to 0^+$, and the case $\Delta t\to 0^-$ is similar by writing $-\Delta t=(\sqrt{-\Delta t})^2$. A combination of the last two equations yields (2.7), which completes the proof. □

Using Lemma 2.1, we shall establish our major result, the model for stock price dynamics.

Theorem 2.1. *Under the hypotheses (H1)-(H2), the stock price satisfies*

$$dS=\mu S dt+\frac{\hat{v}}{\boldsymbol{V}_0}S dB, \tag{2.9}$$

where $\hat{v}=\hat{v}(t)$ is the trading intensity defined in (2.2), $\boldsymbol{V}_0$ *is the number of free-float shares, and B denotes the standard Brownian motion.*

Proof. First, replacing t_0 with t and replacing t with $t+\Delta t$ in (2.5) of hypothesis (H1), we have

$$S(t+\Delta t)=S(t)e^{\mu\Delta t}+\frac{1}{\boldsymbol{V}_0}\int_t^{t+\Delta t}[S(t+\Delta t)-S(\tau)]v(\tau)d\tau, \tag{2.10}$$

which, by subtracting $S(t)$ and then dividing Δt to both sides, gives

$$\frac{S(t+\Delta t)-S(t)}{\Delta t}=S(t)\frac{e^{\mu\Delta t}-1}{\Delta t}+\frac{1}{\boldsymbol{V}_0\Delta t}\int_t^{t+\Delta t}\big(S(t+\Delta t)-S(\tau)\big)v(\tau)d\tau.$$

Then, sending $\Delta t \to 0$ and making use of Lemma 2.1, we get

$$\frac{dS(t)}{dt} = \mu S(t) + \frac{\hat{v}}{\boldsymbol{V}_0} S(t) \frac{dB(t)}{dt},$$

or equivalently

$$dS = \mu S dt + \frac{\hat{v}}{\boldsymbol{V}_0} S dB,$$

competing the proof. □

Compared with the GBM (1.1), in the model process (2.9), the volatility σ is replaced with the trading volume, which is another stochastic process. In this regard, our model is similar to stochastic volatility models, and the difference lies in replacing the volatility σ with the volume v, a much easier measurable quantity.

By the Itô calculus

$$d\ln S(t) = \frac{dS(t)}{S(t)} - \frac{d\langle S(t), S(t)\rangle}{2S^2(t)} = \frac{dS(t)}{S(t)} - \frac{\hat{v}^2(t)}{2\boldsymbol{V}_0^2} dt,$$

substituting this into (2.9), we get

$$d\ln S(t) = \left(\mu - \frac{\hat{v}^2(t)}{2\boldsymbol{V}_0^2}\right) dt + \frac{\hat{v}(t)}{\boldsymbol{V}_0} dB(t). \tag{2.11}$$

Integrating the above equation on $[0,T]$ and taking the exponential to the resulting equation, we obtain

$$S(T) = S(0) \exp\left\{ \int_0^T \left(\mu - \frac{\hat{v}^2(t)}{2\boldsymbol{V}_0^2}\right) dt + \int_0^T \frac{\hat{v}(t)}{\boldsymbol{V}_0} dB(t) \right\}. \tag{2.12}$$

Remark 2.1. When $\hat{v}(t)/\boldsymbol{V}_0 := \sigma$ is a constant, the above price equation (2.12) reduces to the classical GBM

$$S(T) = S(0) \exp\left\{ \left(\mu - \frac{\sigma^2}{2}\right) T + \sigma B(T) \right\}, \tag{2.13}$$

thus the classical GBM model could be regarded as a special case of our model.

3 Applications to option pricing

An accurate description of stock price dynamics is the fundamental basis in the valuation of options. The widely used BS model assumes the stock price follows the GBM

$$dS = \mu S dt + \sigma S dB, \tag{3.1}$$

where the volatility σ is considered as a constant. Based on this hypothesis and the Delta-hedge strategy, Black-Scholes-Merton [4, 6, 12, 15] shows the option value obeys a parabolic equation

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0, \tag{3.2}$$

where $V=V(S,t)$ is the price of an option writing on the stock S. The value of a European call (resp. put) option can then be determined uniquely with the terminal condition $V(S,T)=\max\{S-K,0\}$ (resp. $V(S,T)=\max\{0,K-S\}$). Here, T is the expiry date and K is the strike price. Therefore, $V=V(S,t;r,\sigma;K,T)$, and when fitting the predicted price with the actual trading price in market, it seems the only adjustable parameter is the volatility: $\sigma=\sigma(V,S,t;r;K,T)$ can be solved by the inverse problem and is termed by the implied volatility. The famous volatility smile and the term structure indicate the implied volatility is not a constant, which contradicts the basic assumption of the BS model. Many other models are proposed to accommodate this market feature – the volatility should not be a constant. The local and stochastic volatility models are two mainly used kinds among them. Dupire [7] proposed the local volatility model to resolve the volatility smile, where he assumed $\sigma=\sigma(S,t)$ is a function of the stock price and time. On the other hand, the stochastic volatility model considers $\sigma=\sigma(t)$ follows a stochastic process, for instance, the celebrated Heston model [11] assumes that $\sigma^2(t)$ follows a CIR process

$$d\sigma^2(t)=\kappa\big(\theta-\sigma^2(t)\big)dt+\xi\sigma(t)d\tilde{B}(t), \tag{3.3}$$

where θ is the long-run average of the volatility, κ is the reverting rate, ξ is the volatility of volatility, and $\tilde{B}(t)$ is another Brownian motion that is correlated with $B(t)$.

In our description of the stock price dynamics, trading volume plays a vital role instead of volatility. Inspired by the previous works, we may develop a local volume model as Dupire did and a stochastic volume model as Heston did for the option value. For instance, if the stock price follows

$$dS(t)=rS(t)dt+\frac{\hat{v}(t)}{\boldsymbol{V}_0}S(t)dB(t), \tag{3.4}$$

$$d\hat{v}^2(t)=\kappa\big(\theta-\hat{v}^2(t)\big)dt+\xi\hat{v}(t)d\tilde{B}(t). \tag{3.5}$$

The option price $V=V(S,\hat{v},t)$ satisfies a two-dimensional parabolic PDE. The theoretical analysis, numerical methods and empirical studies for such models will benefit the option pricing research and practice.

In our study, we have developed a general framework for pricing stocks based on trading volume dynamics without specifying a particular model for trading volumes. Note that our framework is not restricted to any specific choice of trading volume models. For instance, alternative models like jump-diffusion processes or those with regime-switching structures could also be considered. This flexibility in our approach allows for the exploration of various trading volume models and their potential impact on stock pricing.

Appendix A. Hypothesis testing for (H2)

Our central hypothesis (H2) states that the price's instantaneous rate of change (ROC) follows a white noise process, as shown in (2.6). This appendix presents an empirical study and verification of this hypothesis via hypothesis testing based on actual market data.

The data used in this paper are from actual A-share transactions, including 50 stocks from 2016 to 2017, which are obtained from the Wind Financial Terminal and are not disclosed here due to privacy policies. We will use graphs and hypothesis tests to examine the Gaussian and white noise properties of relative rates. Specifically, we apply the Shapiro-Wilk test [17] and the Kolmogorov-Smirnov test [8] to investigate the Gaussian property and use the Q-statistic to test for the autocorrelation. The details and the results of the experiments are presented below. The price rate of change (ROC) is given as

$$\mathrm{ROC}=\frac{S(t+\Delta t)-S(t)}{S(t)}, \tag{A.1}$$

where Δt is a small time interval with $\Delta t=1500,2000,3600,4000,5000,6000,7200,9000,$ 14400 seconds, and the time t is chosen as $t=k\Delta t, k=0,1,2,3,\ldots.$ Noting that the unit of time in finance is one year, thus the above time interval can be regarded as small, for instance, $\Delta t=3600\mathrm{s}=1/1000$ if 250 trading days annually and 4 trading hours per day.

Fig. 1 presents the histogram of the ROC for Vanke (000002.SZ) from 2016 to 2017. From the histograms shown in Fig. 1, we may conjecture that the ROC sequence satisfies the properties of normality or white noise. To obtain the general law of stocks, we use a box diagram to remove outliers from the ROC, following the basic steps of statistical testing. We present the box diagram for the Vanke example in Fig. 2.

Then we test the Gaussian and white noise properties of the ROC. We employ the S-W and K-S tests, commonly used in statistics to test Gaussian properties. Table 1 presents the hypothesis test results of Vanke (000002.SZ) from 2016 to 2017. Both tests reject the null at the 5% level for the ROC of Vanke. Hence, it is reasonable to conclude that the ROC does not satisfy Gaussian properties. We further check the white noise properties of the ROC. The top panels of Figs. 3-11 present a sequential chart of the ROC, the middle panels present its autocorrelation chart, and the bottom panels are partial autocorrelation diagrams. Figs. 3-11 use different Δt to draw pictures with delay order $\log(n)+1$ and sample size n. The sequential charts in the top panels of Figs. 3-11 show that the ROC fluctuates around 0, and the upper and lower distributions are uniform. In addition, the autocorrelation and partial autocorrelation coefficients after order 0 are within the upper and lower boundaries of the blue confidence interval, so the ROC can be considered white noise with 95% confidence. Barlett proved that if a time series is purely random, for a sequence with m observation periods, the autocorrelation coefficient of the delay non-zero order sample of the sequence will approximately obey a normal distribution with mean of zero and variance of $1/m$

$$\hat{\rho}_k\sim\mathcal{N}(0,1/m),\quad k=1,2,3,\ldots.$$

Table 1: Hypothesis test of Gaussian property about the difference quotient for Vanke from 2016 to 2017.

Δt (s)	p-value (S-W)	p-value (K-S)
1500	1.96E-12	0
2000	3.27E-10	0
3600	1.32E-06	2.51E-270
4000	1.37E-06	9.63E-241
5000	2.46E-06	3.07E-192
6000	1.96E-05	9.17E-160
7200	8.43E-05	2.83E-134
9000	0.000219571	1.89E-107
14400	0.040090751	2.46E-66

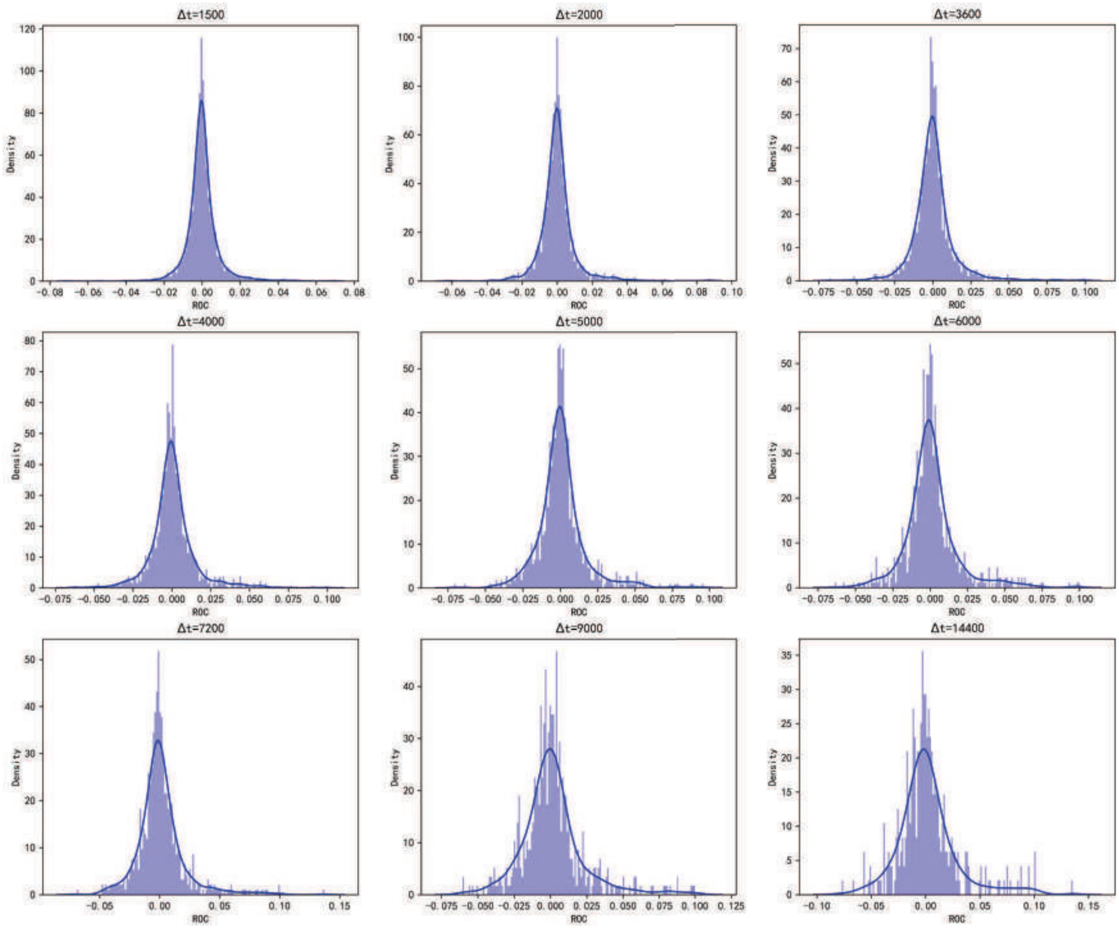

Figure 1: Histogram of the ROC for Vanke from 2016 to 2017.

Table 2: The Q-statistic about the difference quotient for Vanke.

lag	Δt (s)								
	1500	2000	3600	4000	5000	6000	7200	9000	14400
1	0.0370	0.0898	0.0712	0.2643	0.5299	0.9131	0.1913	0.4476	0.2347
2	0.0838	0.0937	0.0986	0.4438	0.7221	0.1906	0.1285	0.5173	0.4073
3	0.1506	0.1663	0.1223	0.5741	0.8092	0.3124	0.1654	0.6213	0.5742
4	0.2247	0.2606	0.0864	0.6042	0.8008	0.4207	0.2462	0.6754	0.6987
5	0.2346	0.2654	0.1409	0.4498	0.7988	0.4307	0.3412	0.5787	0.4660
6	0.3258	0.3348	0.1609	0.0777	0.8625	0.4009	0.3901	0.6915	0.2682
7	0.2965	0.4357	0.1680	0.1191	0.9182	0.4821	0.4958	0.7009	
8	0.3457	0.4996	0.1051						

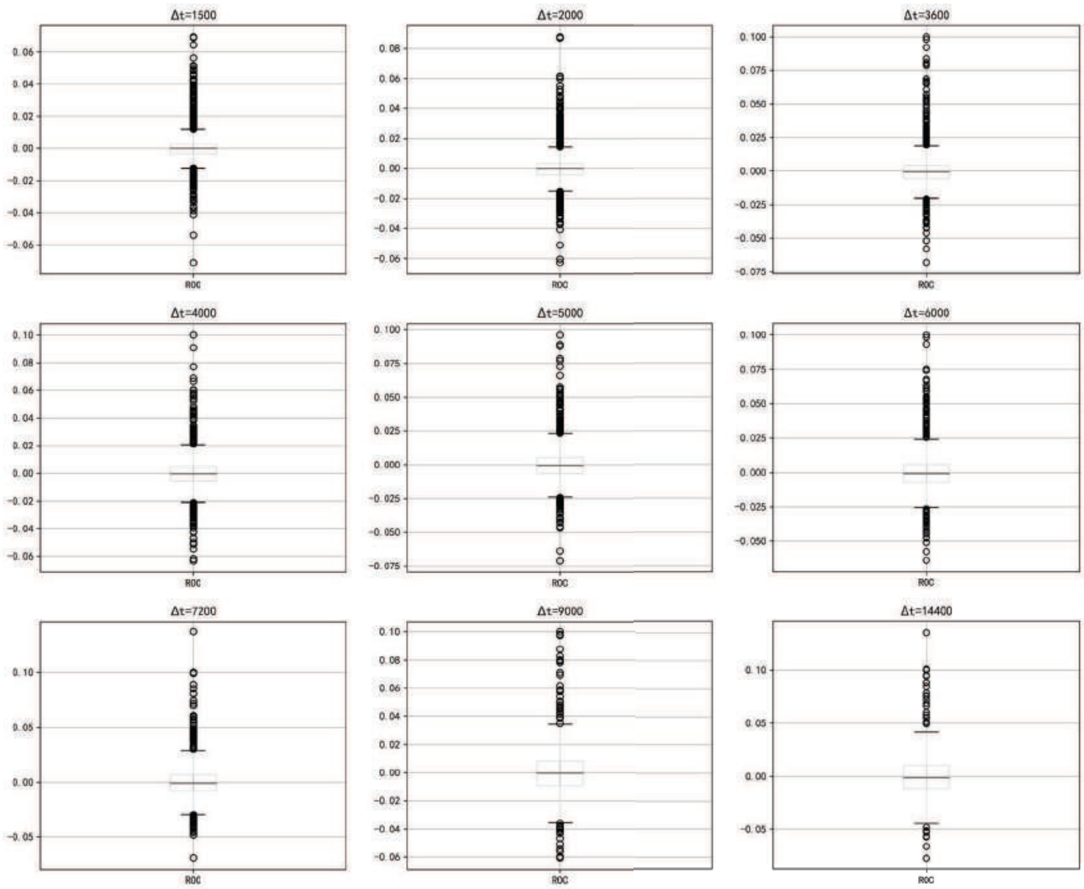

Figure 2: Box diagram of the ROC for Vanke from 2016 to 2017.

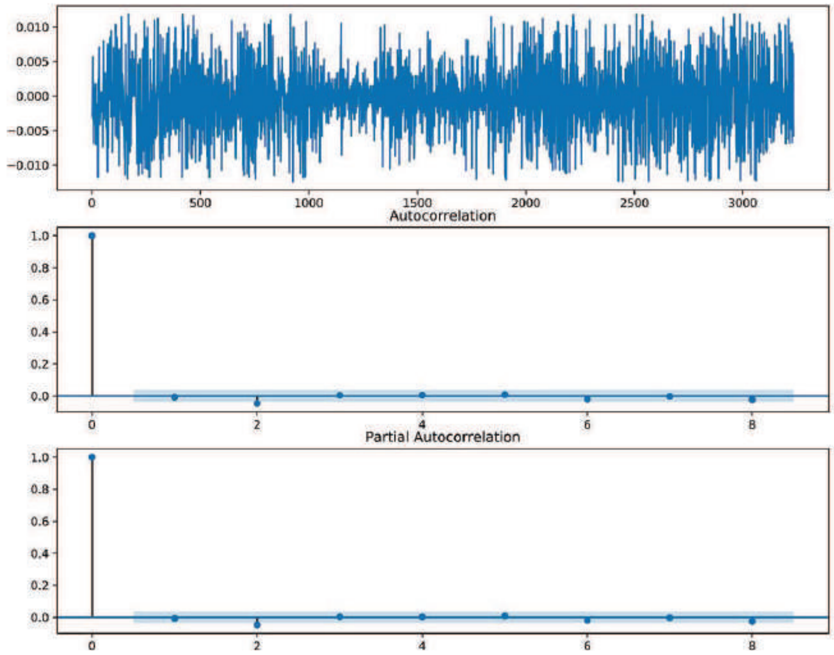

Figure 3: Δt=1500s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

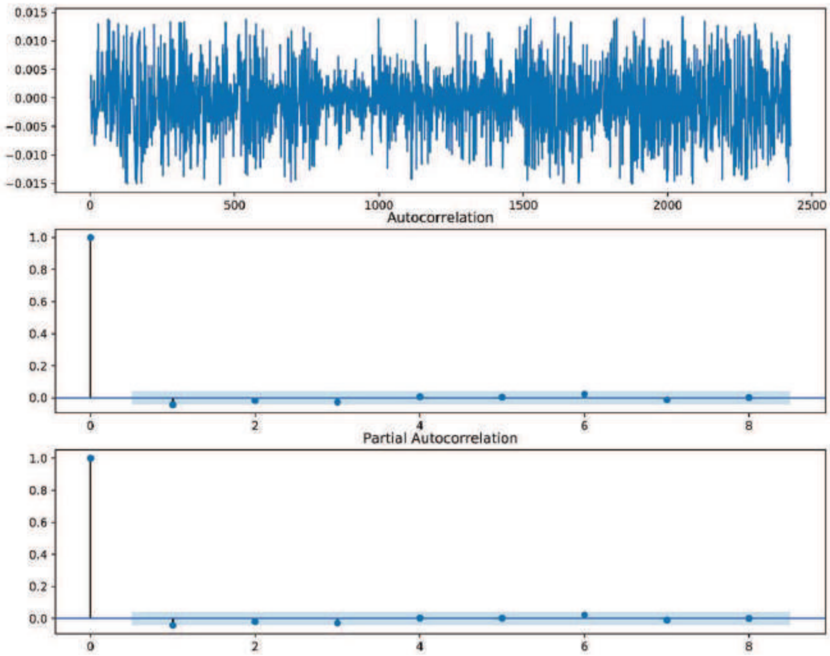

Figure 4: Δt=2000s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

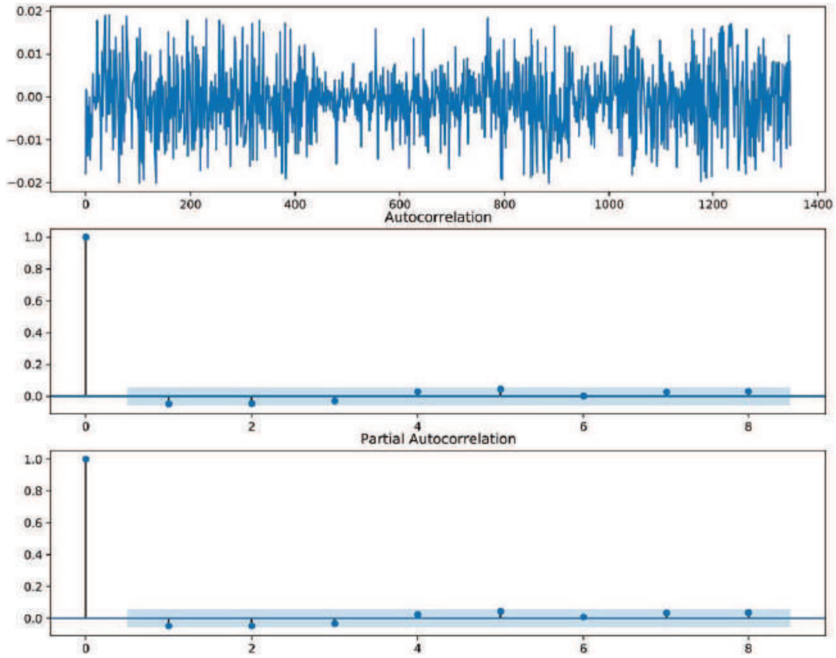

Figure 5: Δt=3600s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

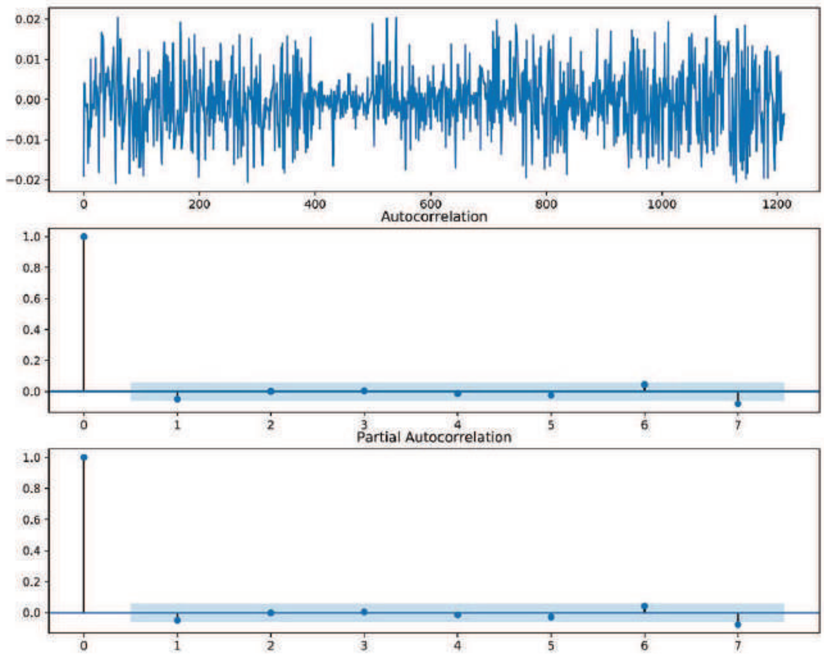

Figure 6: Δt=4000s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

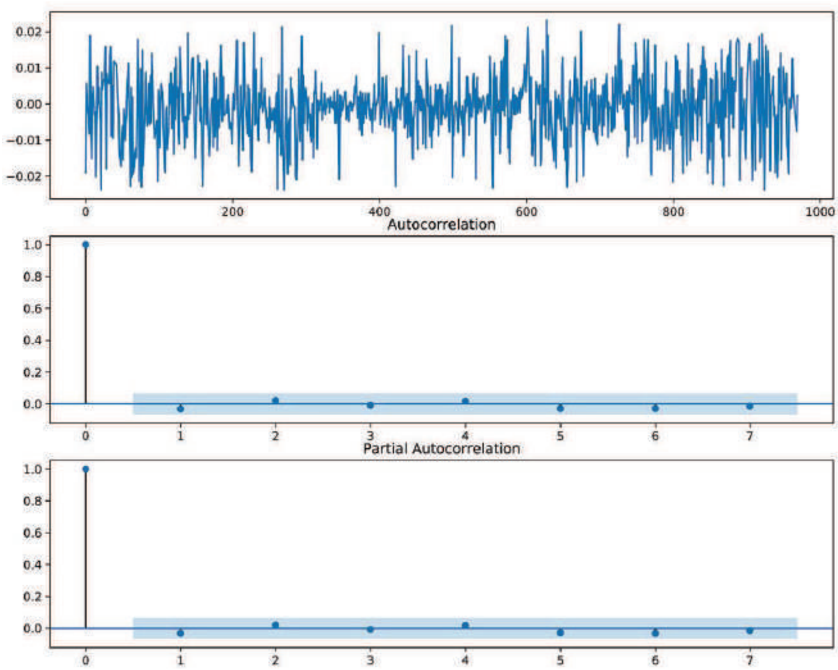

Figure 7: Δt=5000s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

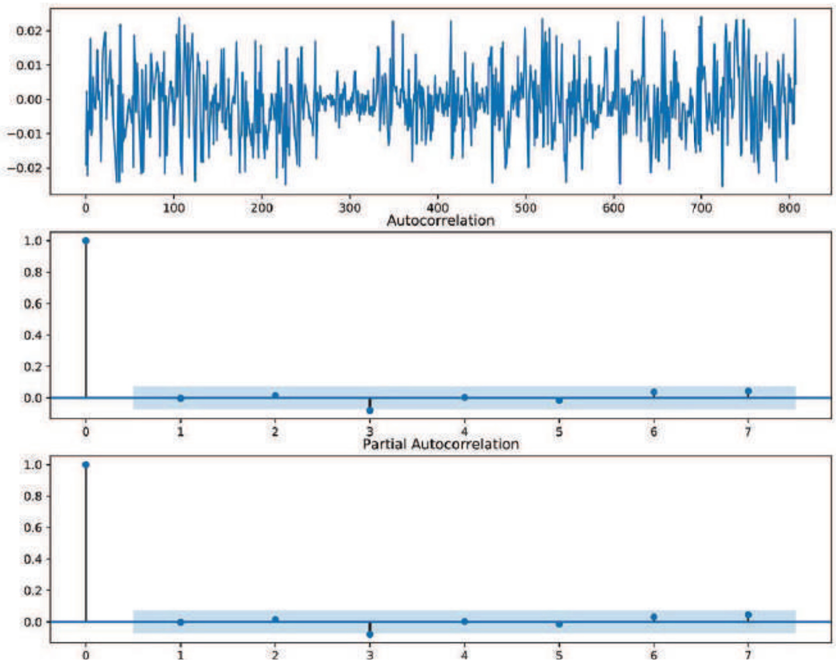

Figure 8: Δt=6000s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

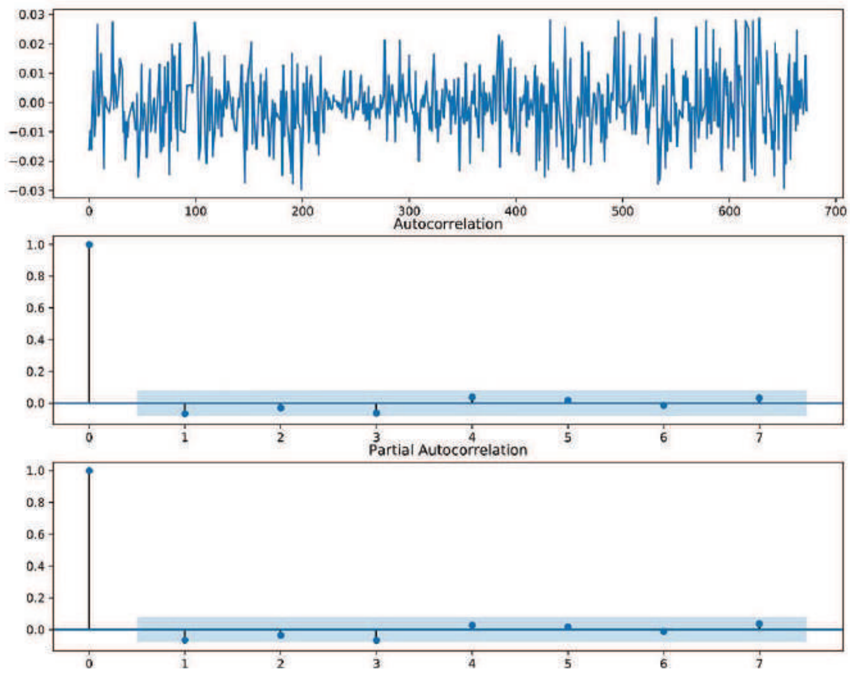

Figure 9: Δt=7200s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

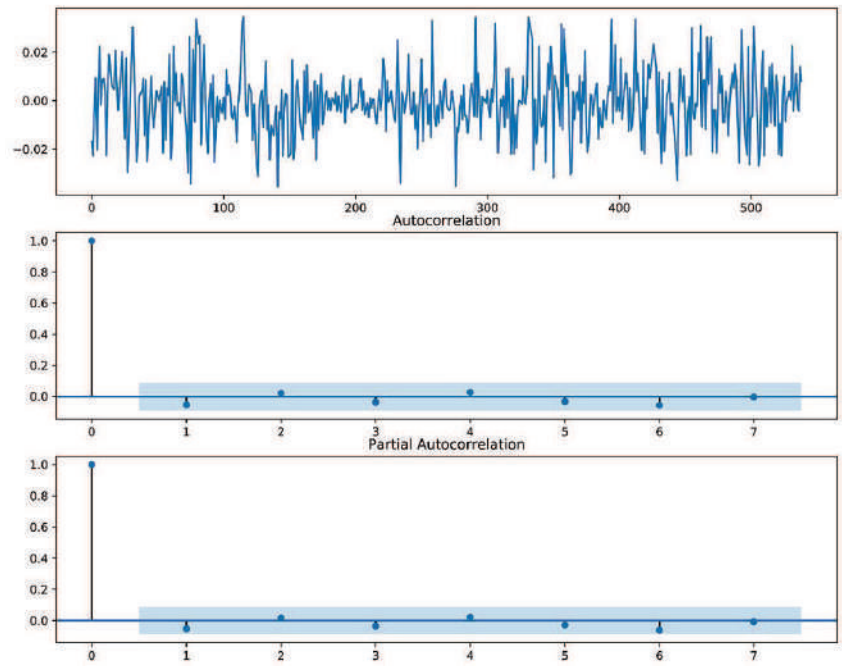

Figure 10: Δt=9000s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

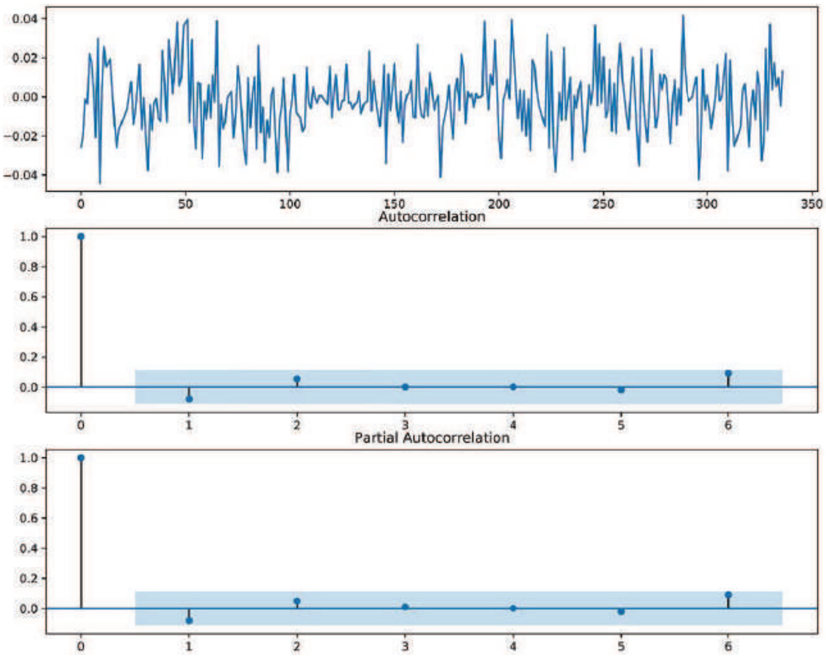

Figure 11: Δt=14400s. Top: Sequential chart of the ROC. Middle: Autocorrelation chart. Bottom: Partial autocorrelation diagrams.

Based on the above argumentation, Box and Pierce [5] derived the Q-statistic for checking the pure randomness of sequences. Table 2 presents the hypothesis test results of the Q-statistic for Vanke (000002.SZ), and we draw similar conclusions to the graphical test. All H_0 with different Δt can not be rejected at the 1% level and can be accepted at the 5% level except for $\Delta t = 1500s$ and lag $= 1$. Therefore, both graphical and hypothesis test results confirm that the ROC is a white-noise sequence.

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References

[1] T. G. Andersen, *Return volatility and trading volume: An information flow interpretation of stochastic volatility*, J. Finance, 51(1):169–204, 1996.

[2] L. Bachelier, *Théorie de la spéculation*, Annales Scientifiques de l'École Normale Supérieure, 3(17):21–86, 1900.

[3] H. Bessembinder and P. J. Seguin, *Price volatility, trading volume, and market depth: Evidence from futures markets*, J. Financial Quant. Anal., 28(1):21–39, 1993.

[4] F. Black and M. Scholes, *The pricing of options and corporate liabilities*, J. Polit. Econ., 81(3):637–654, 1973.

[5] G. E. P. Box and D. A. Pierce, *Distribution of residual autocorrelations in autoregressive-integrated moving average time series models*, J. Amer. Statist. Assoc., 65:1509–1526, 1970.

[6] R. H. Chan, Y. Z. Guo, S. T. Lee, and X. Li, *Financial Mathematics, Derivatives and Structured Products*, Springer, 2019.

[7] B. Dupire, *Pricing with a smile*, Risk, 7(1):18–20, 1994.

[8] J. Durbin, *Distribution Theory for Tests Based on the Sample Distribution Function*, SIAM, 1973.

[9] E. F. Fama, *The behavior of stock-market prices*, The Journal of Business, 38:34–105, 1965.

[10] C. W. J. Granger and O. Morgenstern, *Spectral analysis of New York stock market prices*, Kyklos, 16(1):1–27, 1963.

[11] S. L. Heston, *A closed-form solution for options with stochastic volatility with applications to bond and currency options*, Rev. Financ. Stud., 6:327–343, 1993.

[12] J. Hull, *Options, Futures, and Other Derivatives*, Pearson, 2022.

[13] C. M. Jones, G. Kaul, and M. L. Lipson, *Transactions, volume, and volatility*, Rev. Financ. Stud., 7:631–651, 1994.

[14] B. B. Mandelbrot, *The variation of certain speculative prices*, The Journal of Business, 36:394–419, 1963.

[15] R. Merton, *Theory of rational option pricing*, Bell J. Econ. Manage. Sci., 4:141–183, 1973.

[16] P. Samuelson, *Rational theory of warrant pricing*, Ind. Manage. Rev., 6(2):13–32, 1965.

[17] S. S. Shapiro and M. B. Wilk, *An analysis of variance test for normality (complete samples)*, Biometrika, 52:591–611, 1965.

[18] X. E. Xu, P. Chen, and C. Wu, *Time and dynamic volume-volatility relation*, J. Bank. Financ., 30:1535–1558, 2006.